

Chapter 9

Design of Flat Slabs

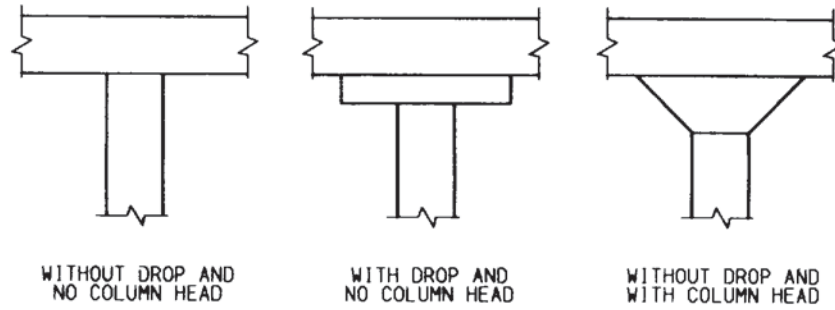
9.0 NOTATION

A	Area of column or area of effective column head
A_{st}	Area of steel in tension
b_e	Effective width of slab for transfer of moment to edge column
C_x	Size of a rectangular column in x -direction
C_y	Size of a rectangular column in y -direction
d	Effective depth of tensile reinforcement
d_h	Depth of column head
f_y	Characteristic yield strength of reinforcement
f_{cu}	Characteristic cube strength of concrete at 28 days
h_c	Effective diameter of column or effective column head
G_k	Characteristic dead load
l_c	Dimension of column in direction of l_h
l_h	Effective dimension of column head
l_x	Shorter span framing onto columns
l_y	Longer span framing onto columns
l_{ho}	Actual dimension of column head
$l_{h,max}$	Maximum dimension of column head taking 45° dispersion
l_1	Centre-to-centre of column in direction of span being considered
l_2	Centre-to-centre of column perpendicular to direction of span being considered
M'	Design limit moment at $h_c/2$
M_t	Moment transferred to column by frame analysis
$M_{t,max}$	Limiting moment between flat slab and edge column
n	Total ultimate load per unit area on flat slab
Q_k	Characteristic live load
V_t	Calculated shear from analysis
V_{eff}	Effective shear at column/slab interface
W_k	Characteristic wind loading
x	Length of side of a perimeter parallel to axis of bending

9.1 DEFINITIONS

Flat slab is a reinforced concrete slab supported by columns with, or without, drops. The columns may be with, or without, column heads.

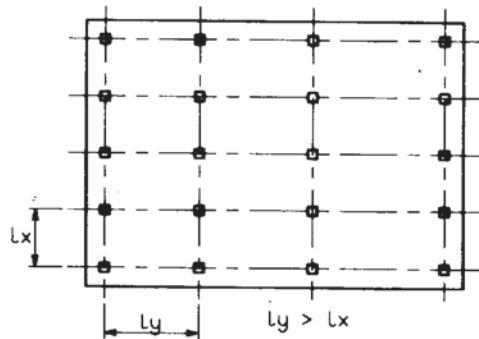
Drop is a local thickening of the slab in the region of the column.



SK 9/1 Flat slab – section. SK 9/2 Flat slab – section. SK 9/3 Flat slab – section.

Column head is a local enlargement of the column at the junction with the slab.

9.2 ANALYSIS OF FLAT SLABS



SK 9/4 Typical plan of flat slab.

Flat slabs are usually supported by a rectangular arrangement of columns. The analysis may be carried out by an equivalent frame method or by the use of a finite element computer code. When using the equivalent frame method the ratio of the longer to the shorter span should not exceed 2. The analysis for uniformly distributed vertical load may be carried out by using Tables 9.1 to 9.6.

The properties of the flat slab for analysis are similar to those already discussed for solid slabs in Chapter 3.

9.2.1 Effective dimension of column head

l_h = effective dimension of head

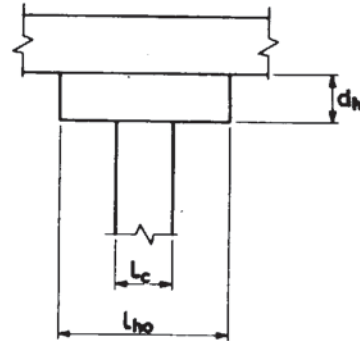
l_{ho} = actual dimension of head

$l_{h,max} = l_c + 2(d_h - 40)$

l_c = dimension of column in direction of l_h

d_h = depth of head

l_h = is taken as the lesser of l_{ho} or $l_{h,max}$



SK 9/5 Flat slab – definitions.

Note: This means that the maximum dimension is limited by a 45° dispersion of column up to 40 mm below the slab.

9.2.2 Effective diameter of a column head

$$h_c = (4A/\pi)^{\frac{1}{2}} \leq 0.25l_x$$

h_c = effective diameter of column or column head

A = area of column or area of effective column head as defined by l_h

l_x = shortest span framing onto column

h_c should not be taken greater than one-quarter of shortest span of slab framing into column.

9.2.3 Drops

Drops will be effective in the analysis if the smaller dimension of the drop is at least one-third of the smallest span of surrounding panels.

For the checking of punching shear, this limitation does not apply.

9.2.4 Load combinations for analysis

$$LC_1 = 1.4G_k + 1.6Q_k \quad \text{on all spans}$$

$$LC_2 = 1.4G_k + 1.6Q_k \quad \text{on alternate spans and other spans loaded with } 1.0G_k$$

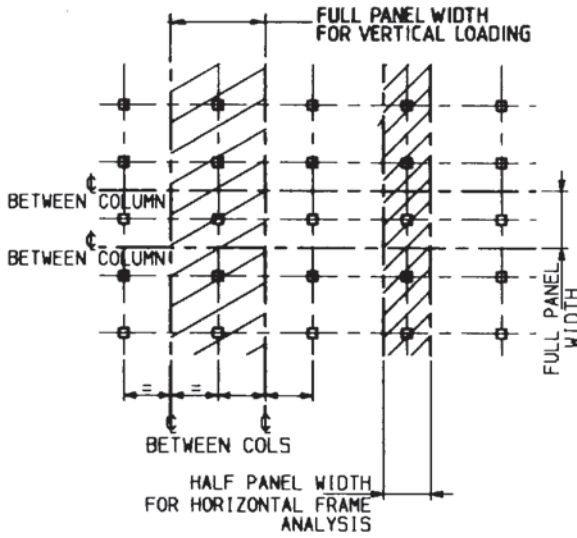
where G_k = characteristic dead load

Q_k = characteristic live load.

9.2.5 Effective width of slab for analysis

For vertical loading assume full width of panel between columns for frame analysis.

For horizontal loading as a frame assume stiffness of half width of panel.



SK 9/6 Plan of flat slab showing panel widths for analysis.

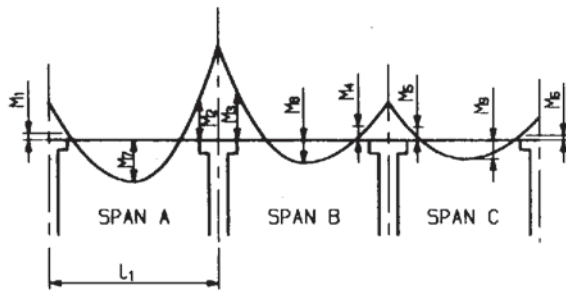
The analysis should be carried out using a computer program or a moment distribution method. The analysis may also be carried out for uniformly distributed vertical loads using Tables 9.1 to 9.6.

The analysis may be carried out using Table 3.13 of BS8110: Part 1: 1985^[1] provided the lateral stability is not dependent on the slab-column connection and loading on the flat slab for the design is based on a single load case, i.e. LC_1 , the ratio of Q_k/G_k does not exceed 1.25, Q_k does not exceed 5 kN/m^2 , and there are at least three rows of panels.

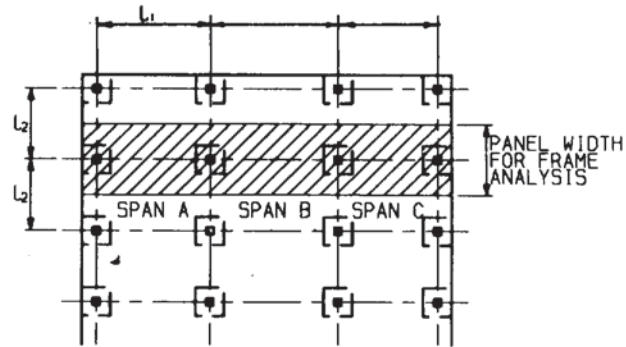
9.3 DESIGN OF FLAT SLABS

The design may be based on the negative moment at $h_c/2$ from the centreline of the column. But this negative moment will have to be modified if the sum of the positive design moment and the average negative design moment is less than the following expression:

$$M' = \left(\frac{nl_2}{8}\right)\left(l_1 - \frac{2h_c}{3}\right)^2$$



SK 9/7 Negative moment limitation for flat slabs - section.



SK 9/8 Typical plan of flat slab – negative moment limitation.

where l_1 = centre-to-centre of column in direction of span being considered
 l_2 = centre-to-centre of column perpendicular to direction of span being considered
 n = total ultimate load on slab (kN/m^2).

To give an example:

For Span A

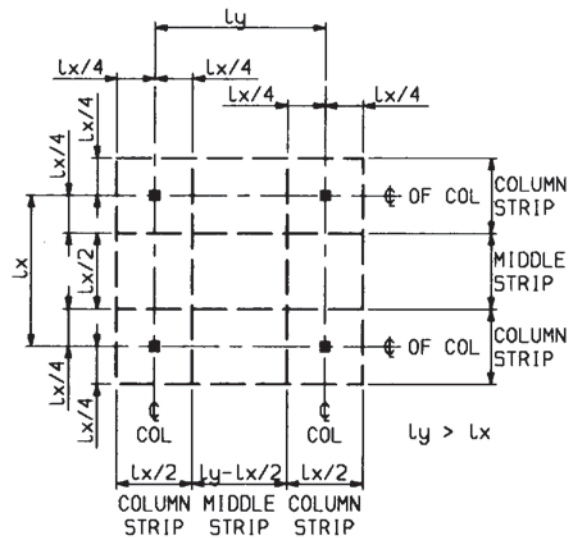
$$0.5 (M_1 + M_2) + M_7 \geq M'$$

For Span B

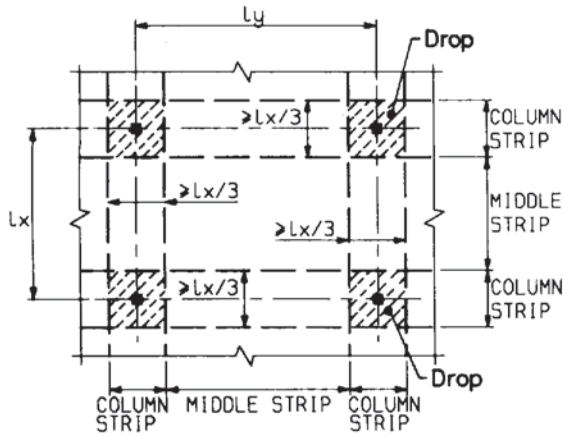
$$0.5(M_3 + M_4) + M_7 \geq M'$$

Increase negative moments M_1, M_2, M_3 , etc. until these conditions are satisfied.

9.3.1 Division of panels



SK 9/9 Flat slab – division of strips.



PLAN OF SLAB WITH DROP

IGNORE DROP IF DROP WIDTH < $l_x/3$

SK 9/10 Flat slab – division of strips.

Panels are divided into column strips and middle strips as shown.

For slab without drop the column strip is $l_x/4$ wide on either side of the centreline of column, where l_x is the shorter span.

For slab with drop the column strip is the size of the drop. Ignore drop if the size of the drop is less than $l_x/3$.

9.3.2 Division of moments between columns and middle strips

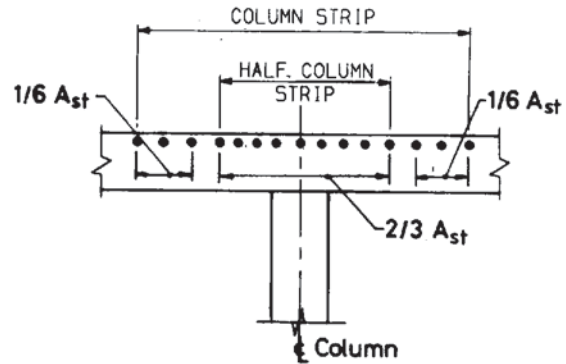
The moments obtained from analysis of frames should be divided as follows (these percentages are for slabs without drops):

	Column strip	Middle strip
Negative	75%	25%
Positive	55%	45%

Note: Where column drops are used and column strips are determined from the width of the drop, it may so happen that the middle strip is bigger than the middle strip in a slab without drop. In that case the moments in the middle strip will be proportionately increased and those in the column strip decreased to keep the total positive and negative moment unchanged.

9.3.3 Design of flat slab panels

The design is similar to the design of slabs and the worked examples are in Chapter 3.

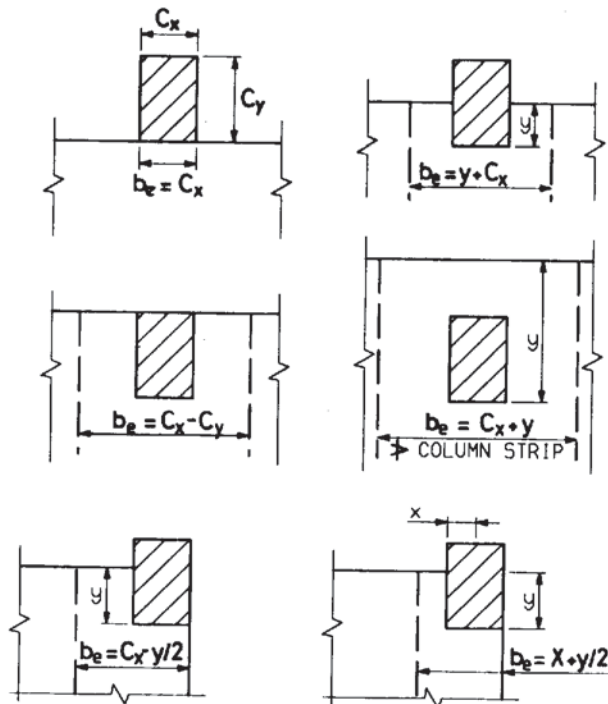


SK 9/11 Detailing of reinforcement in flat slabs.

Internal panels and edge panels

Two-thirds of the negative support reinforcement in the column strip should be placed in half the width of the column strip centred over the column.

9.3.4 Moment connection to edge column



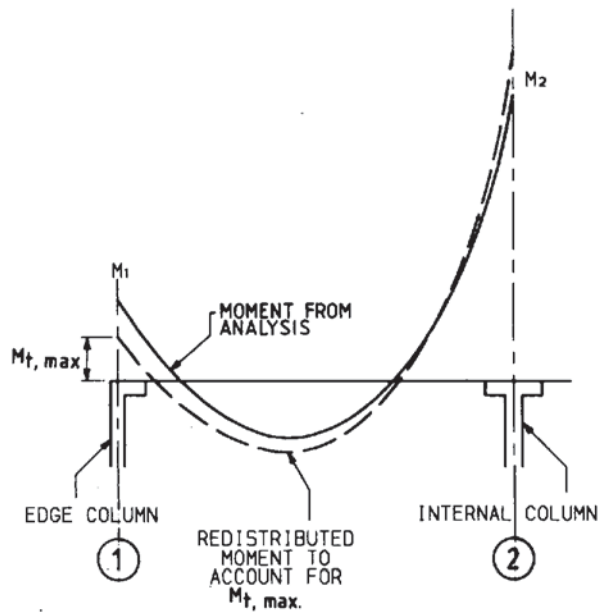
SK 9/12 Effective width of slab for moment connection to edge column.

See sketches above to find effective width of slab b_e for transfer of moment between flat slab and edge column. This moment should be limited to

$$M_{t,max} = 0.15b_e d^2 f_{cu}$$

where d = effective depth of top reinforcement in column strip.

The moment $M_{t,max}$ should not be less than half the design moment from an equivalent frame analysis or 70% of the design moment from a grillage or finite element analysis. The structural arrangement may be changed if $M_{t,max}$ does not satisfy the above condition.



SK 9/13 Insufficient moment transfer capacity at edge column.

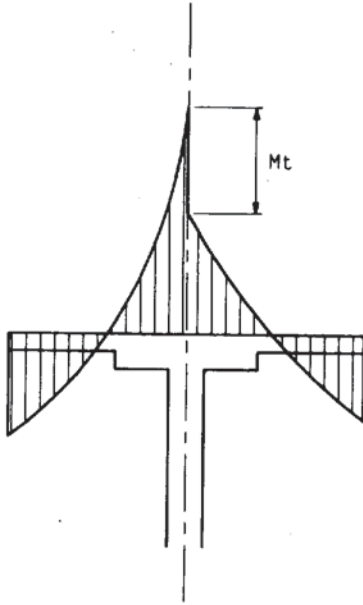
Where the design moment is larger than $M_{t,max}$, redistribution of moment may be carried out to reduce the design moment to $M_{t,max}$. Otherwise, to transfer moments in excess of $M_{t,max}$ to edge column, the edge of the slab should be reinforced by an edge beam or an edge strip. The edge beam will be designed to carry the additional moment by torsion to the column.

9.3.5 Shear in flat slabs

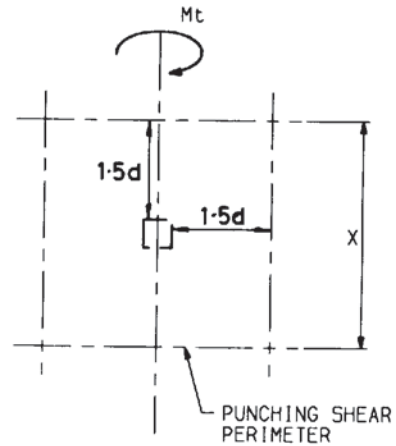
Punching shear around columns should be checked according to Step 7 of Section 3.3. The shear to be considered for the punching shear calculation is increased from the calculated column shear by an amount dependent on the moment transferred to the column by frame action.

For internal column connections,

$$V_{eff} = V_t \left(1 + \frac{1.5M_t}{V_t x} \right)$$



SK 9/14 Moment diagram at an internal column of a flat slab.



SK 9/15 Definition of dimension x .

where V_t = calculated shear from analysis
 M_t = moment transferred to column by frame analysis
 x = length of side of perimeter considered parallel to axis of bending.

Alternatively,

$$V_{\text{eff}} = 1.15V_t \quad \text{for simplicity}$$

For corner column connections,

$$V_{\text{eff}} = 1.25V_t$$

For edge column connections,

$$V_{\text{eff}} = 1.25V_t \quad \text{for bending about axis parallel to free edge}$$

$$V_{\text{eff}} = V_t \left(1.25 + \frac{1.5M_t}{V_t x} \right) \quad \text{for bending about axis perpendicular to free edge}$$

Alternatively,

$$V_{\text{eff}} = 1.4V_t$$

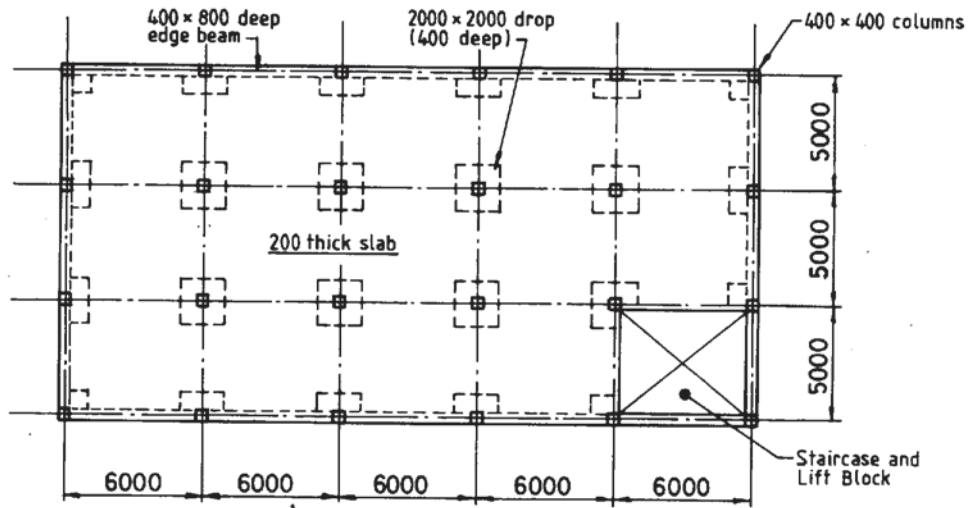
The moment M_t may be reduced by 30% where the equivalent frame analysis is used and both load cases LC_1 and LC_2 have been considered. The shear reinforcement will be calculated according to Step 7 of Section 3.3.

9.4 STEP-BY-STEP DESIGN PROCEDURE FOR FLAT SLABS

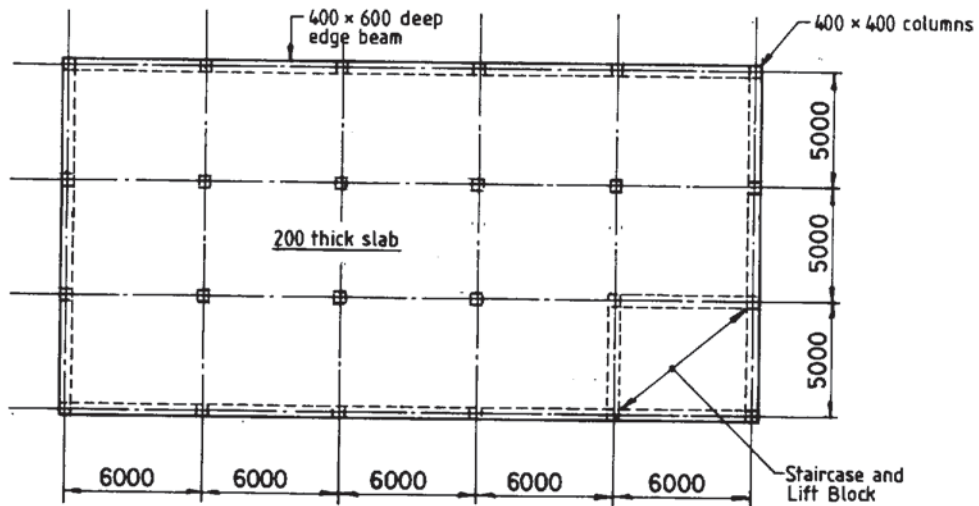
- Step 1* Carry out analysis as in Section 9.2.
- Step 2* Find moment connection to edge column as per Section 9.3.4 and redistribute moments if necessary.
- Step 3* Draw bending moment diagrams and calculate moments at $h_c/2$ following Section 9.3.
- Step 4* Check limitation of negative design moments following Section 9.3.
- Step 5* Carry out division of panels as in Section 9.3.1.
- Step 6* Divide moments between column strips and middle strips as per Section 9.3.2.
- Step 7* Determine cover to reinforcement (see Step 3 of Section 3.3).
- Step 8* Carry out design for flexure as per Step 4 of Section 3.3.
- Step 9* Distribute reinforcement as per Section 9.3.3.
- Step 10* **Check punching shear stress**
Follow Step 7 of Section 3.3.
- Step 11* **Check span/effective depth ratio**
Follow Step 11 of Section 3.3 for slabs with drops. For slabs without drops follow the same step but multiply l_c/d from Table 11.3 by 0.9.
- Step 12* **Curtailement of bars**
Follow Step 12 of Section 3.3.
- Step 13* **Spacing of bars**
Follow Step 13 of Section 3.3.
- Step 14* **Check early thermal cracking**
Follow Step 14 of Section 3.3.
- Step 15* **Calculate minimum reinforcement**
Follow Step 9 and Step 15 of Section 3.3.
- Step 16* **Calculate flexural crack width**
Follow Step 16 of Section 3.3.
- Step 17* **Design of connections**
Follow Chapter 11.

9.5 WORKED EXAMPLE

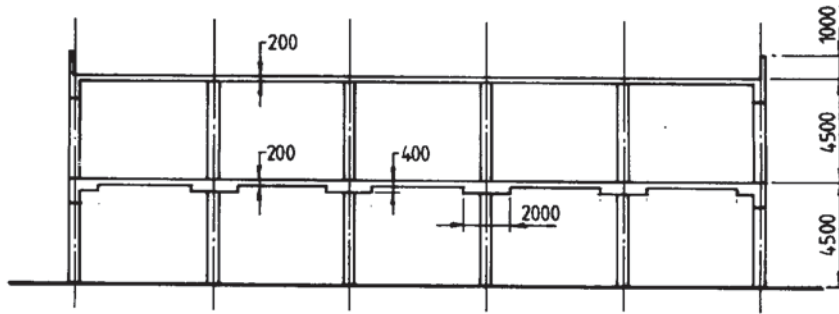
Example 9.1 Flat slab construction for a sports hall



SK 9/16 Plan on first floor.



SK 9/17 Plan on roof.



SK 9/18 Section through building.

Two-storey building plan size: 15 m × 30 m

Column grid: 5 m × 6 m

Column size: 400 × 400

Height of building = 10 m overall

Topography factor, $S_1 = 1.0$

Ground roughness factor, $S_2 = 0.95$

Statistical factor for wind, $S_3 = 1.0$

Basic wind speed = 42 m/s = V

Design wind speed = $S_1 S_2 S_3 V = 40$ m/s

$q = k V_s^2 = 1$ kN/m²

$C_{pe} = +0.7$ and -0.3

$C_{pi} = -0.3$ (four faces equally permeable)

The above wind pressure coefficients are obtained from CP3: Chapter V *Wind loads*.^[14]

Live load on roof = 1.5 kN/m²

Live load on floor = 5 kN/m²

Floor slab has 2000 × 2000 drop at columns

Thickness of roof and floor slab = 200 mm

Thickness at drop of floor slab = 400 mm

Continuous perimeter edge beam 400 wide × 800 deep

Centre-to-centre height of floor = 4.5 m

Step 1 Carry out analysis

Only one frame in the short direction of the building will be analysed.

Column head has not been used.

Effective diameter of column, $h_c = \left(\frac{4A}{\pi}\right)^{\frac{1}{2}} \leq 0.25l_x$

$A = 400 \times 400 = 160\,000$ mm²

$$\begin{aligned} \therefore h_c &= \left(\frac{4 \times 160\,000}{\pi}\right)^{\frac{1}{2}} \\ &= 451 \text{ mm} < 0.25l_x \end{aligned}$$

$l_x = 5000$ mm

$0.25l_x = 1250$ mm

Drop of 2000 mm in floor slab is greater than $l_x/3 = 1667$ mm.
Drop will be effective in the distribution of moment.

Loading

Frames in short direction are 6 m apart.

Roof slab

$$G_k = \text{characteristic dead load} = 0.2 \times 25 = 5 \text{ kN/m}^2$$

$$Q_k = 1.5 \text{ kN/m}^2$$

$$LC_1 = 1.4G_k + 1.6Q_k = 9.4 \text{ kN/m}^2 = 56.4 \text{ kN/m}$$

$$LC_2 = 9.4 \text{ kN/m}^2 \text{ and } 5 \text{ kN/m}^2 \text{ on alternate spans}$$

$$\text{or } LC_2 = 56.4 \text{ kN/m} \text{ and } 30 \text{ kN/m} \text{ on alternate spans}$$

Floor slab

$$G_k = 5 \text{ kN/m}^2 \text{ at slab without drop}$$

$$= 10 \text{ kN/m}^2 \text{ at slab with drop (area } 2 \text{ m} \times 2 \text{ m)}$$

$$= 30 \text{ kN/m} \text{ or } 30 + 5 \times 2 = 40 \text{ kN/m}$$

$$Q_k = 5 \text{ kN/m}^2$$

$$= 30 \text{ kN/m}$$

$$LC_1 = 1.4G_k + 1.6Q_k$$

$$LC_2 = \text{alternate spans loaded with } LC_1 \text{ and dead load only}$$

Columns

Horizontal load on columns is due to wind load at the rate of 1 kN/m^2 which is equivalent to 6 kN/m on the column. The wind loading analysis will be carried out separately and combined later with the vertical loading because the stiffness of the slab to resist horizontal loading is half of that to resist vertical loading.

Load cases with wind load W_k are as follows:

$$LC_3 = 1.4G_k + 1.4W_k$$

$$LC_4 = 1.2G_k + 1.2Q_k + 1.2W_k$$

Frame analysis using a computer software

$$E = \text{Young's modulus} = 28 \times 10^6 \text{ kN/m}^2$$

12 joints 14 members

Joints 1, 4, 7 and 10 rigidly fixed.

Column size 400×400

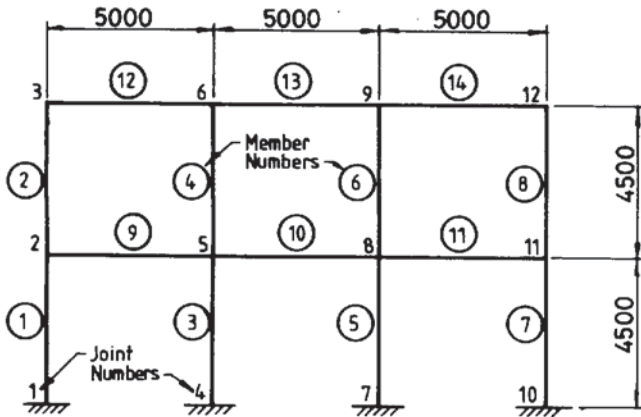
Slab size 6000×200 (deep)

Load cases:

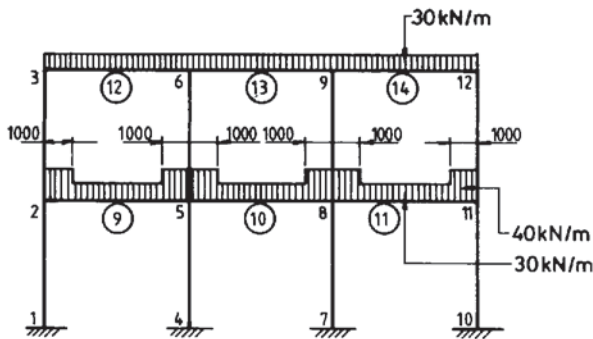
B_1 – dead load

B_2 to B_7 – live loads on members 9 to 14 respectively

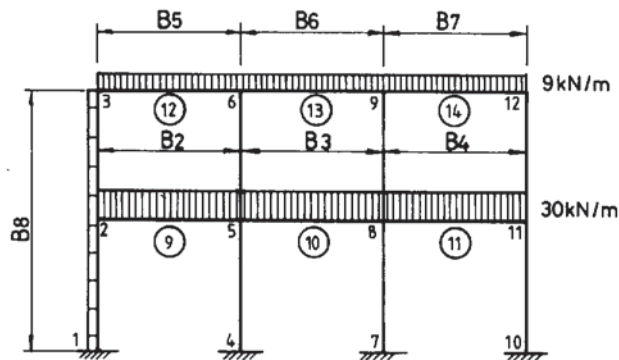
B_8 – wind load



SK 9/19 Frame diagram for analysis.



SK 9/20 Dead load on frame (B_1).



SK 9/21 Basic live loads B_2 to B_8 .

Combinations:

$$C_1 = 1.4B_1 + 1.6(B_2 + B_3 + B_4 + B_5 + B_6 + B_7)$$

$$C_2 = 1.4B_1 + 1.6(B_2 + B_4 + B_5 + B_7)$$

$$C_3 = 1.4B_1 + 1.6(B_3 + B_6)$$

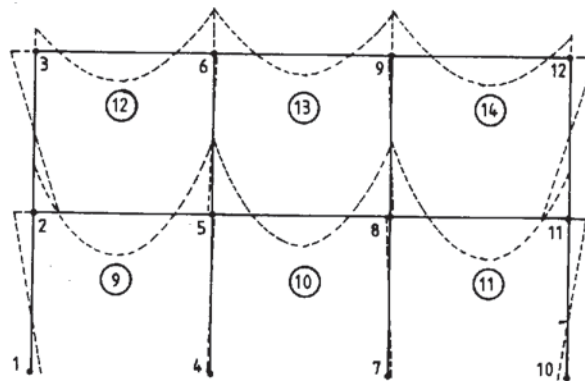
$$C_4 = 1.4B_1 + 1.6(B_2 + B_3 + B_4)$$

Output from analysis

Envelope of load cases (vertical loads)

Elastic analysis – no redistribution

Floor slab	Member 9			
Joint	Maximum BM	Shear	Combination	
2	130.80	228.5	C ₂	
5	215.5	258.2	C ₄	
Midspan	131.2	—	C ₂	
Floor slab	Member 10			
Joint	Maximum BM	Shear	Combination	
5	199.9	239.0	C ₄	
8	199.9	239.0	C ₄	
Midspan	112.5	—	C ₃	
Roof slab	Member 12			
Joint	Maximum BM	Shear	Combination	
3	67.8	130.0	C ₂	
6	129.0	153.5	C ₁	
Midspan	82.0	—	C ₂	
Roof slab	Member 13			
Joint	Maximum BM	Shear	Combination	
6	121.3	141.0	C ₁	
9	121.3	141.0	C ₁	
Midspan	60.4	—	C ₃	



SK 9/22 Combination C₁ – bending moment diagram.

Envelope of load cases (vertical + horizontal loads) (The analysis of horizontal load is carried out with half stiffness of slab)

Elastic analysis – no redistribution

Combinations:

$$C_5 = 1.4B_1 + 1.4B_8$$

$$C_6 = 1.2B_1 + 1.2(B_2 + B_3 + B_4 + B_5 + B_6 + B_7) + 1.2B_8$$

Floor slab	Member 9		
Joint	Maximum BM	Shear	Combination
2	121.9	185.9	C_6
5	191.4	215.2	C_6
Midspan	95.8	—	C_6
Floor slab	Member 10		
Joint	Maximum BM	Shear	Combination
5	176.0	198.6	C_6
8	176.0	198.6	C_6
Midspan	71.4	—	C_6
Roof slab	Member 12		
Joint	Maximum BM	Shear	Combination
3	57.0	108	C_6
6	112.2	128.9	C_6
Midspan	67.1	—	C_6
Roof slab	Member 13		
Joint	Maximum BM	Shear	Combination
6	108.1	119.8	C_6
9	108.1	119.8	C_6
Midspan	45.4	—	C_6

Carry out redistribution of moment:

Maximum bending moment at joint 5 = 215.5 kNm

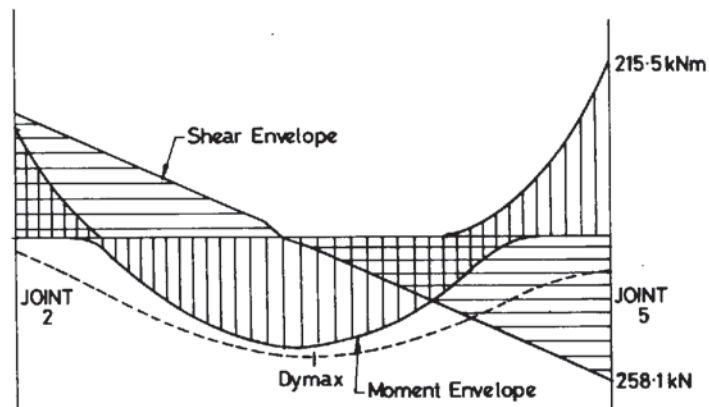
Assume 20% redistribution.

Set plastic moment capacity at joint 5 = $0.8 \times 215.5 = 172.4$ kNm

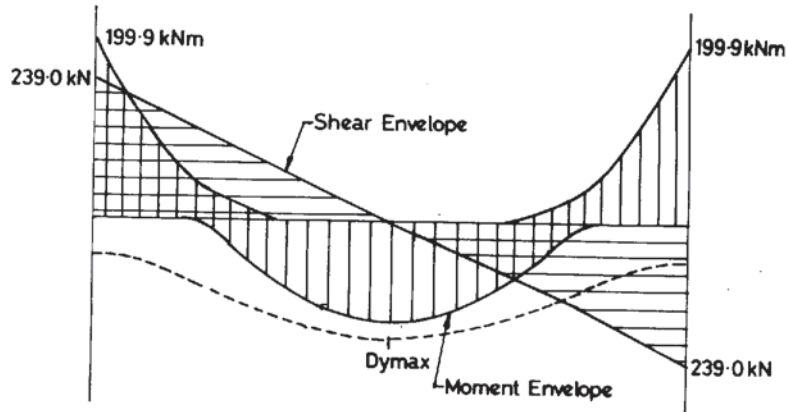
Similarly

maximum bending moment at joint 6 = 129 kNm

Assume 20% redistribution.



SK 9/23 Shear and moment envelope for member 9.



SK 9/24 Shear and moment envelope for member 10.

Set plastic moment capacity at joint 6 = $0.8 \times 129 = 103.2 \text{ kNm}$

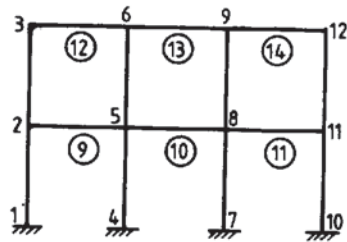
The following steps of reanalysis of frame are carried out:

- Step 1: For one combination at a time increase live load on span until plastic moment is reached at a joint in a member. Plastic moment capacity of members on first floor is 172.4 kNm and member on roof is 103.2 kNm.
- Step 2: Release joint where plastic moment is reached and increase loading until plastic moment capacity is reached at another joint.
- Step 3: Progressively release joints and increase live load until full complement of live load is on structure.
- Step 4: Find cumulative effect of all incremental live load on structure.

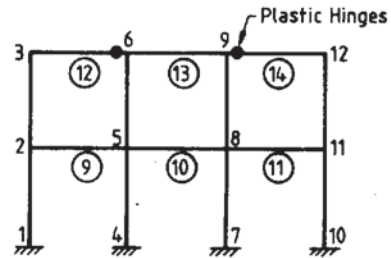
The following tables become useful if a non-linear finite element computer package is not available.

Frame types:

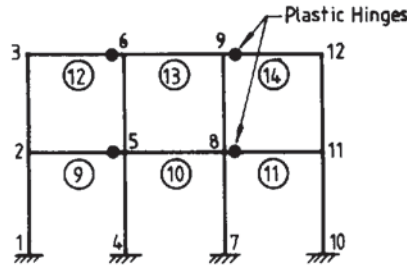
- F_1 = no member end releases
- F_2 = member in F_1 nos 12 and 14 ends released at joints 6 and 9
- F_3 = member in F_2 nos 9 and 11 ends released at joints 5 and 8
- F_4 = member in F_3 no. 13 ends released at joints 6 and 9
- F_5 = member in F_4 no. 10 ends released at joints 5 and 8



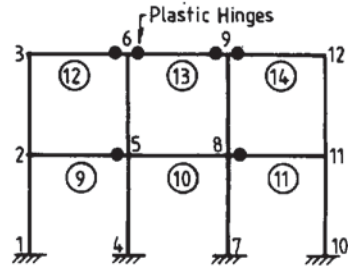
SK 9/25 Frame type F_1 .



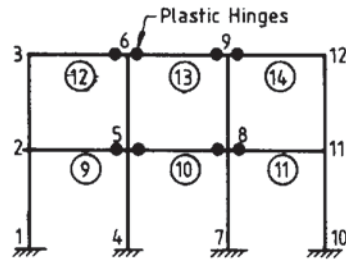
SK 9/26 Frame type F_2 .



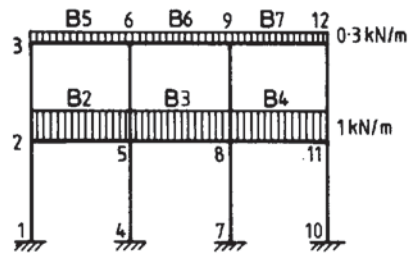
SK 9/27 Frame type F_3 .



SK 9/28 Frame type F_4 .



SK 9/29 Frame type F_5 .



SK 9/30 Unit live load on frame.

Combination $C_7 = 1.4 \times \text{dead load}$ or $1.4B_1$

The method is illustrated for combinations C'_1 and C'_2 only.

$$C'_1 = 1.6(B'_2 + B'_3 + B'_4 + B'_5 + B'_6 + B'_7)$$

$$C'_2 = 1.6(B'_2 + B'_4 + B'_5 + B'_7)$$

$$B'_2, B'_3, B'_4 = 1 \text{ kN/m}$$

$$B'_5, B'_6, B'_7 = 0.3 \text{ kN/m}$$

Combination	Frame type	Member end bending moments (kNm)							
		Member 9		Member 10		Member 12		Member 13	
		Joint 2	Joint 5	Joint 5	Joint 8	Joint 3	Joint 6	Joint 6	Joint 9
C_7	F_1	62.7	102.5	95.5	95.5	44.7	97.5	91.6	91.6
C'_1	F_1	2.0	3.7	3.4	3.4	0.7	1.1	1.0	1.0
C'_1	F_2	2.1	3.6	3.5	3.5	0.9	0	0.5	0.5
C'_1	F_3	3.1	0	2.4	2.4	1.1	0	0.8	0.8
C'_1	F_4	3.1	0	2.2	2.2	1.1	0	0	0
C'_1	F_5	3.1	0	0	0	1.1	0	0	0
C'_2	F_1	2.3	2.7	0.8	0.8	0.8	0.8	0.2	0.2
C'_2	F_2	2.3	2.6	0.8	0.8	0.9	0	-0.2	-0.2
C'_2	F_3	3.1	0	0	0	1.0	0	0	0
C'_2	F_4	3.1	0	0	0	1.0	0	0	0
C'_2	F_5	3.1	0	0	0	1.0	0	0	0

$C_1 = 1$ unit of live load combination in combination C_1 i.e. $C_1 = 1$ kN/m of B_2, B_3 and B_4 and $9/30$ kN/m of B_5, B_6 and B_7 .

Full compliment of B_2, B_3 and B_4 is 30 kN/m and of B_5, B_6 and B_7 is 9 kN/m.

Plastic moment at joint 5 is fixed at 172 kNm

Plastic moment at joint 6 is fixed at 103.2 kNm

Dead load moment at joint 6 = 97.5 kNm

Each unit of combination C_1 produces 1.1 kNm at joint 6 for frame type F_1 . Therefore

units of live load in combination C_1 required to form first plastic hinges at joint 6 and joint 9 in members 12 and 14

$$= \frac{103.2 - 97.5}{1.1} = 5 \text{ units, say}$$

Frame type F_2 has joints released at joints 6 and 9 for members 12 and 14. After 5 units of combination C_1 the bending moments at joints are as follows:

Frame type F_1

Member 9	Joint 2	$62.7 + 5 \times 2.0 = 72.7$ kNm
	Joint 5	$102.5 + 5 \times 3.7 = 121.0$ kNm
Member 10	Joint 5	$95.5 + 5 \times 3.4 = 112.5$ kNm
	Joint 3	$44.7 + 5 \times 0.7 = 48.2$ kNm
Member 12	Joint 6	$97.5 + 5 \times 1.1 = 103$ kNm *plastic
	Joint 6	$91.6 + 5 \times 1.0 = 96.6$ kNm

Units of live load in combination C_1 to form second plastic hinges at joints 5 and 8 in members 9 and 11

$$= \frac{172 - 121}{3.6} = 14 \text{ units of combination } C_1$$

Total number of units of C_1 to cause plastic hinges at joints 5 and 8 in members 9 and 11 is 19.

After 19 units of combination C_1 , the bending moments at joints are as follows:

Frame type F_2

Member 9	Joint 2	$72.7 + 14 \times 2.1 = 102.1$ kNm
	Joint 5	$121 + 14 \times 3.6 = 171.4$ kNm *plastic
Member 10	Joint 5	$112.5 + 14 \times 3.5 = 161.5$ kNm
	Joint 3	$48.2 + 14 \times 0.9 = 60.8$ kNm
Member 12	Joint 6	plastic = 103 kNm *plastic
	Joint 6	$96.6 + 14 \times 0.5 = 103.6$ kNm *plastic

Joint 5 of member 9 and joint 6 of member 13 have gone plastic simultaneously at 19 units of combination C_1 . Therefore frame type F_3 is not considered.

After 24 units of combination C_1 , the bending moments at joints are as follows:

Frame type F_4

Member 9	Joint 2	$102.1 + 5 \times 3.1 = 117.6 \text{ kNm}$	
	Joint 5	plastic	$= 171.4 \text{ kNm}$ *plastic
Member 10	Joint 5	$161.5 + 5 \times 2.2 = 172.5 \text{ kNm}$	*plastic
Member 12	Joint 3	$60.8 + 5 \times 1.1 = 66.3 \text{ kNm}$	
	Joint 6	plastic	$= 103 \text{ kNm}$ *plastic
Member 13	Joint 6	plastic	$= 103.6 \text{ kNm}$ *plastic

Frame type F_5

After 30 units of combination C_1 , the bending moments at joints are as follows:

Member 9	Joint 2	$117.6 + 6 \times 3.1 = 136.2 \text{ kNm}$	
	Joint 5		$= 171.4 \text{ kNm}$
Member 10	Joint 5		$= 172.5 \text{ kNm}$
Member 12	Joint 3	$66.3 + 6 \times 1.1 = 72.9 \text{ kNm}$	
	Joint 6		$= 103.0 \text{ kNm}$
Member 13	Joint 6		$= 103.6 \text{ kNm}$

Formula for calculating midspan bending moment and shear

$C_7 + 5$ units of $C'_1 (F_1) + 14$ units of $C'_1 (F_2) + 5$ units of $C'_1 (F_4) + 6$ units of $C'_1 (F_5)$

Member 9 – combination C_1 (20% redistribution)

$$\text{Midspan moment} = 56.3 + 2.2 \times 5 + 2.2 \times 14 + 5 \times 3.5 + 6 \times 3.5 = 136.6 \text{ kNm}$$

$$\text{End shear, joint 2} = 111.1 + 3.7 \times 5 + 3.7 \times 14 + 4.6 \times 5 + 4.6 \times 6 = 232.0 \text{ kN}$$

$$\text{End shear, joint 5} = 126.9 + 4.3 \times 5 + 4.3 \times 14 + 3.4 \times 5 + 3.4 \times 6 = 246.0 \text{ kN}$$

Similarly

Member 10 – combination C_1 (20% redistribution)

$$\text{Midspan moment} = 115.7 \text{ kNm}$$

$$\text{End shear} = 239 \text{ kN}$$

Member 12 – combination C_1 (20% redistribution)

$$\text{Midspan moment} = 90.9 \text{ kNm}$$

$$\text{End shear 3} = 134.9 \text{ kN}$$

$$\text{End shear 6} = 147.1 \text{ kN}$$

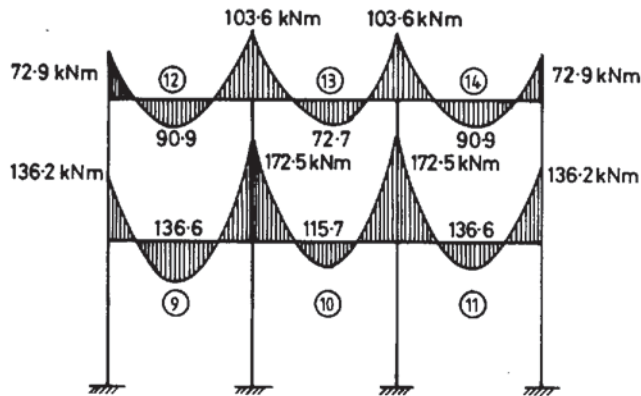
Member 13 – combination C_1 (20% redistribution)

$$\text{Midspan moment} = 72.7 \text{ kNm}$$

$$\text{End shear 6} = 141 \text{ kN}$$

From computer output:
Member midspan moments (kNm) and shears (kN) for various frame types under unit loading.

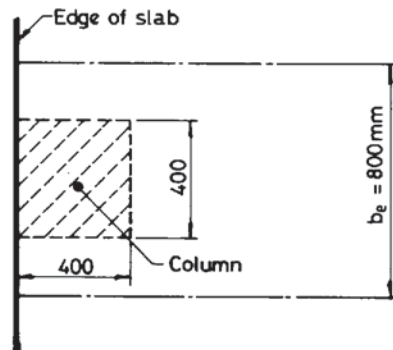
Combination	Frame type	Member 9		Member 10		Member 12		Member 13					
		Midspan moment	Shear	Midspan moment	Shear	Midspan moment	Shear	Midspan moment	Shear				
C7	F1	56.3	111.1	126.9	42.7	119.0	119.0	61.5	94.4	115.6	39.7	105.0	105.0
C1	F1	2.2	3.7	4.3	1.6	4.0	4.0	0.6	1.1	1.3	0.5	1.2	1.2
C1	F2	2.2	3.7	4.3	1.5	4.0	4.0	1.1	1.4	1.0	1.0	1.2	1.2
C1	F3	3.5	4.6	3.4	2.6	4.0	4.0	1.0	1.4	1.0	0.7	1.2	1.2
C1	F4	3.5	4.6	3.4	2.8	4.0	4.0	1.0	1.4	1.0	1.5	1.2	1.2
C1	F5	3.5	4.6	3.4	5.0	4.0	4.0	1.0	1.4	1.0	1.5	1.2	1.2



SK 9/31 Bending moment diagram combination C_1 (20% redistribution).

Note: Only one combination C_1 has been fully analysed to demonstrate the procedure for redistribution of moments in a frame structure. In practice all combinations of loads should be similarly processed to get an envelope of moments and shears. For all combinations of loads the plastic hinges will form at the same moment, i.e. 172 kNm at first floor level and 103.2 kNm at roof level.

Step 2 Check moment connection to edge column



SK 9/32 Effective width of slab for moment transfer.

$$M_{t, \max} = 0.15b_e d^2 f_{cu}$$

$$b_e = C_x + C_y = 400 + 400 = 800 \text{ mm}$$

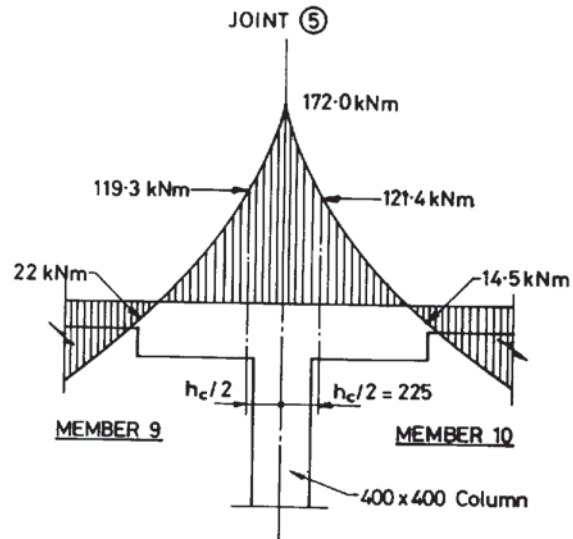
$$d = 175 \text{ mm} \text{ assumed}$$

$$f_{cu} = 40 \text{ N/mm}^2$$

$$\begin{aligned} M_{t, \max} &= 0.15 \times 800 \times 175^2 \times 40 \\ &= 147 \text{ kNm} > 136.2 \text{ kNm} \text{ (member 9, joint 2)} \end{aligned}$$

The column slab connection at the edge can transfer the applied moment and no further redistribution is necessary. It is conservatively assumed in this analysis that the depth of the slab at the column is 200 mm, ignoring the drop. The moment $M_{t, \max}$ is greater than the design moment obtained from an equivalent frame analysis.

Step 3 Find bending moments at $h_c/2$ and at edge of drop



SK 9/33 Bending moments at critical points – combination C_1 (20% redistribution).

Member 9 – combination C_1 (20% redistribution)

Joint 5:

Bending moment = 172 kNm

Shear = 246 kN

Dead load: $1.4 \times 40 = 56$ kN/m near support

Live load: $1.6 \times 30 = 48$ kN/m near support

$h_c/2 = 0.225$ m

$$\begin{aligned} \text{Bending moment at } h_c/2 &= 172 - 246 \times 0.225 + \frac{(56 + 48) \times 0.225^2}{2} \\ &= 119.3 \text{ kNm (top tension)} \end{aligned}$$

Edge of drop = 1000 mm from centreline of column

$$\begin{aligned} \text{Bending moment at edge of drop} &= 172 - 246 \times 1 + \frac{(56 + 48) \times 1^2}{2} \\ &= -22 \text{ kNm (bottom tension)} \end{aligned}$$

Joint 2, similarly:

$$\begin{aligned} \text{Bending moment at } h_c/2 &= 136.2 - 232 \times 0.225 + \frac{(56 + 48) \times 0.225^2}{2} \\ &= 86.6 \text{ kNm (top tension)} \end{aligned}$$

Bending moment at edge of drop = -43.8 kNm (bottom tension)

Member 10 – combination C_1 (20% redistribution)

Joint 5:

Bending moment at $h_c/2 = 121.4 \text{ kNm}$ (top tension)

Bending moment at edge of drop = -14.5 kNm (bottom tension)

Member 12 – combination C_1 (20% redistribution)

Joint 3:

Bending moment at $h_c/2$

$$= 72.9 - 134.9 \times 0.225 + \frac{(1.4 \times 30 + 1.6 \times 9) \times 0.225^2}{2} = 44.0 \text{ kNm}$$

Joint 6:

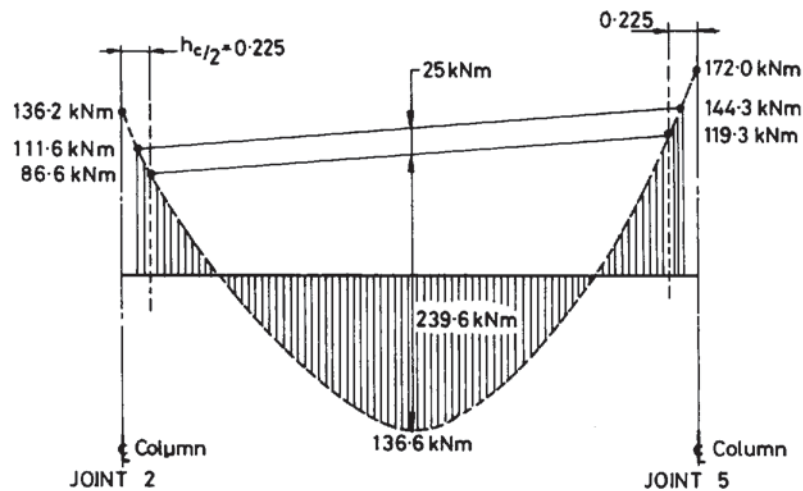
Bending moment at $h_c/2 = 71.3 \text{ kNm}$ (top tension)

Member 13 – combination C_1 (20% redistribution)

Joint 6:

Bending moment at $h_c/2 = 73.3 \text{ kNm}$ (top tension)

Step 4 Check limitation of negative design moment



SK 9/34 Member 9 – combination C_1 (20% redistribution): limitation of negative design moment.

$$M' = \left(\frac{nl_2}{8}\right) \left(l_1 - \frac{2h_c}{3}\right)^2$$

where n = loading per unit area on slab.

Average n on span on first floor = $1.4 \times 5 + 1.6 \times 5 = 15 \text{ kN/m}^2$

$l_1 = 5.0 \text{ m}$ $l_2 = 6.0 \text{ m}$ $h_c = 0.225 \text{ m}$

$$\begin{aligned} M' &= \left(\frac{15 \times 6.0}{8}\right) \left(5.0 - \frac{2 \times 0.225}{3}\right)^2 \\ &= 264.6 \text{ kNm (floor slab)} \end{aligned}$$

Average n on span on roof = $1.4 \times 5 + 1.6 \times 1.5 = 9.4 \text{ kN/m}^2$

$$M' = \left(\frac{9.4 \times 6.0}{8} \right) \left(5.0 - \frac{2 \times 0.225}{3} \right)^2$$

$$= 165.8 \text{ kNm (roof slab)}$$

Check negative moment limitation

Member 9

Joint 2 at $h_c/2 = M_2 = 86.6 \text{ kNm}$ (see Step 3)

Joint 5 at $h_c/2 = M_5 = 119.3 \text{ kNm}$

Midspan moment = 136.6 kNm (see Step 1)

Average of M_2 and M_5 plus midspan moment

$$= 0.5 (86.6 + 119.3) + 136.6$$

$$= 239.6 \text{ kNm} < M' = 264.6 \text{ kNm}$$

The negative moments will have to be increased by $(264.6 - 239.6) = 25.0 \text{ kNm}$

Revised negative moments:

Joint 2: $86.6 + 25.0 = 111.6 \text{ kNm}$

Joint 5: $119.3 + 25.0 = 144.3 \text{ kNm}$

Member 10

Joint 5 at $h_c/2 = M_5 = 121.4 \text{ kNm}$

Midspan moment = 115.7 kNm

Average of negative and positive = $121.4 + 115.7 = 237.1 \text{ kNm} < M' = 264.6 \text{ kNm}$

The negative moments will have to be increased by $(264.6 - 237.1) = 27.5 \text{ kNm}$

Revised negative moments:

Joint 5: $121.4 + 27.5 = 148.9 \text{ kNm}$

Member 12

Joint 3 at $h_c/2 = M_3 = 44.0 \text{ kNm}$

Joint 6 at $h_c/2 = M_6 = 71.3 \text{ kNm}$

Midspan moment = 90.9 kNm

Average of negative and positive = $0.5 (44.0 + 71.3) + 90.9 = 148.55 \text{ kNm} < M' = 165.8 \text{ kNm}$

The negative moments will have to be increased by $(165.8 - 148.6) = 17.2 \text{ kNm}$

Revised negative moments:

Joint 3: $44.0 + 17.2 = 61.2 \text{ kNm}$

Joint 6: $71.3 + 17.2 = 88.5 \text{ kNm}$

Member 13

Joint 6 at $h_c/2 = M_6 = 73.3 \text{ kNm}$

Midspan moment = 72.7 kNm

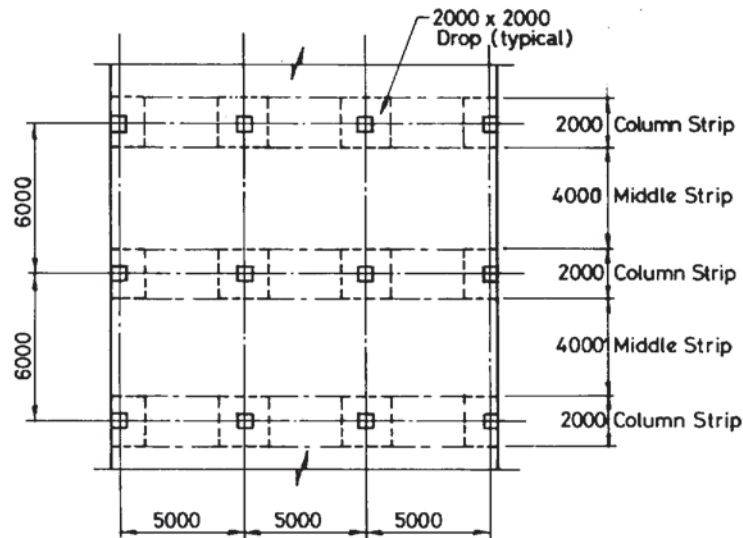
Average of negative and positive = $73.3 + 72.7 = 146 \text{ kNm} < M' = 165.8 \text{ kNm}$

The negative moments will have to be increased by $(165.8 - 146) = 19.8 \text{ kNm}$

Revised negative moments:

Joint 6: $73.3 + 19.8 = 93.1 \text{ kNm}$

Step 5 Carry out division of panels



SK 9/35 Plan of floor slab – division of strips.

First floor slab

Size of drop = 2000 mm

$$\frac{l_x}{3} = \frac{5000}{3} = 1667 \text{ mm} < 2000 \text{ mm}$$

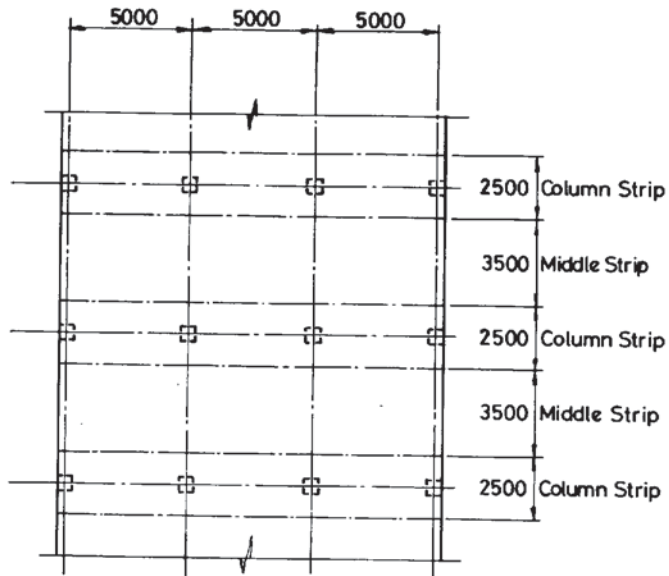
Column strip = 2000 mm

With drop middle strip = $5000 - 2000 = 3000 \text{ mm}$ (y-y)
and $= 6000 - 2000 = 4000 \text{ mm}$ (x-x)

Middle strip in a slab without drop = $6000 - l_x/2 = 6000 - 2500 = 3500 \text{ mm}$

Proportion of middle strip with drop and without drop = $\frac{4000}{3500} = 1.14$

Roof slab



SK 9/36 Plan of roof slab – division of strips.

$$\text{Column strip} = \frac{l_x}{2} = \frac{5000}{2} = 2500 \text{ mm}$$

$$\begin{aligned} \text{Middle strip} &= 6000 - 2500 = 3500 \text{ mm} \quad (x-x) \\ &= 5000 - 2500 = 2500 \text{ mm} \quad (y-y) \end{aligned}$$

Step 6 *Divide moments between column strip and middle strip*

For slabs without drops,

Negative moments – 75% column strip
 25% middle strip

Positive moments – 55% column strip
 45% middle strip

Floor slab: design moments**Member 9: negative moments**

Joint 2: 111.6 kNm (see Step 4)

Joint 5: 144.3 kNm (see Step 4)

Middle strip momentsJoint 2: $111.6 \times 0.25 \times 1.14$ (see Step 4) = 31.8 kNm (top tension)Edge of drop = $43.8 \times 0.25 \times 1.14 = 12.5$ kNm (top tension)Joint 5: $144.3 \times 0.25 \times 1.14 = 41.1$ kNm (top tension)Edge of drop = $22.0 \times 0.25 \times 1.14 = 6.3$ kNm (top tension)

Column strip moments

Joint 2: $111.6 - 31.8 = 79.8 \text{ kNm}$ (top tension)

Edge of drop: $43.8 - 12.5 = 31.3 \text{ kNm}$ (top tension)

Joint 5: $144.3 - 41.1 = 103.2 \text{ kNm}$ (top tension)

Edge of drop: $22 - 6.3 = 15.7 \text{ kNm}$ (top tension)

Member 9: positive moments

Design midspan moment = 136.6 kNm (bottom tension)

Middle strip moments

Midspan: $136.6 \times 0.45 \times 1.14 = 70.1 \text{ kNm}$ (bottom tension)

Column strip moments

Midspan: $136.6 - 70.1 = 66.5 \text{ kNm}$ (bottom tension)

Member 10: negative moments

Joint 5: 148.9 kNm (see Step 4)

Middle strip moments

Joint 5: $148.9 \times 0.25 \times 1.14 = 42.4 \text{ kNm}$ (top tension)

Edge of drop: $14.5 \times 0.25 \times 1.14 = 4.1 \text{ kNm}$ (top tension)

Column strip moments

Joint 5: $148.9 - 42.4 = 106.3 \text{ kNm}$ (top tension)

Edge of drop: $14.5 - 4.1 = 10.4 \text{ kNm}$ (top tension)

Member 10: positive moments

Design midspan moment = 115.7 kNm (bottom tension)

Middle strip moments

Midspan: $115.7 \times 0.45 \times 1.14 = 59.4 \text{ kNm}$ (bottom tension)

Column strip moments

Midspan: $115.7 - 59.4 = 56.3 \text{ kNm}$ (bottom tension)

Note: Similarly calculate moments in column strips and middle strips in roof slab.

Step 7 Determine cover to reinforcement

See Step 3 of Section 3.3.

Step 8 Design for flexure

See Step 4 of Section 3.3.

The increased slab thickness at drops may be considered for the determination of reinforcement provided all reinforcement is properly anchored. Check reinforcement also at edge of drop.

Note: In this example the reinforcement is found for the flat slab spanning in the short direction only. Exactly the same method of analysis and design should be used to find the reinforcement in the long direction.

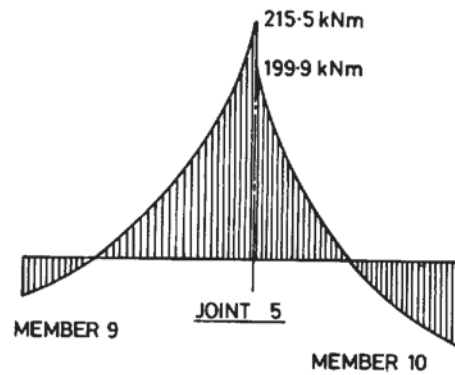
Step 9 Detailing of reinforcement

Two-thirds of the negative support reinforcement in the column strip should be placed in half the width of the column strip centred over the column.

Step 10 Calculate punching shear and shear stress

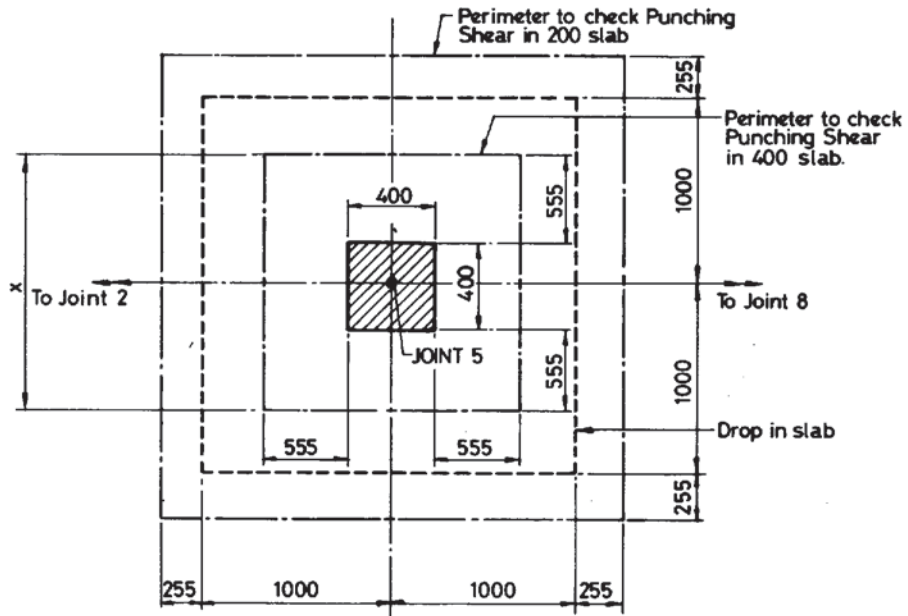
Punching shear at floor slab

Check joint 5.



SK 9/37 Moment transfer to column for punching shear calculation.

BENDING MOMENT DIAGRAM
ELASTIC ANALYSIS - NO REDISTRIBUTION
LOADING CONDITION C4



SK 9/38 Punching shear perimeters – plan of floor slab.

Use results of elastic analysis of frame before redistribution.

Maximum column moment at
joint 5 = 215.5 – 199.9 = 15.6 kNm
 $M_t = 15.6 \text{ kNm}$

A 30% reduction is allowed if frame analysis is carried out.

$$\therefore M_t = 0.7 \times 15.6 = 10.9 \text{ kNm}$$

$$V_t = 258.2 + 239.0 = 497.2 \text{ kN}$$

Punching shear perimeter at $1.5d$ from face of column.

$$\begin{aligned} d &= 400 - 30 = 370 \text{ mm} \\ 1.5d &= 1.5 \times 370 = 555 \text{ mm} \\ x &= 400 + 2 \times 555 = 1510 \text{ mm} = 1.510 \text{ m} \end{aligned}$$

$$\begin{aligned} V_{\text{eff}} &= V_t \left(1 + \frac{1.5M_t}{V_t x} \right) = 497.2 \left(1 + \frac{1.5 \times 10.9}{497.2 \times 1.510} \right) \\ &= 508 \text{ kN} \end{aligned}$$

Maximum shear stress at column perimeter ($U_o = 4 \times 400$)

$$= \frac{V_{\text{eff}}}{U_o d} = \frac{508 \times 10^3}{4 \times 400 \times 370} = 0.86 \text{ N/m}^2 < 5 \text{ N/m}^2 \quad \text{OK}$$

$$\text{Shear stress, } v = \frac{V_{\text{eff}}}{Ud}$$

$$U = 4 \times 1510 = 6040$$

$$\therefore v = \frac{508 \times 10^3}{6040 \times 370} = 0.23 \text{ N/mm}^2 < v_c \quad \text{for minimum percentage of reinforcement}$$

No shear reinforcement is required in slab with drop.

For slab outside drop consider that loaded area is perimeter of drop.

$$\begin{aligned} V_{\text{eff}} &= 508 \text{ kN} - (\text{load on the area of drop}) \\ &= 508 - 4 \times 22 \\ &= 420 \text{ kN} \end{aligned}$$

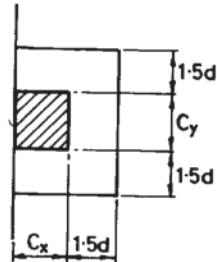
$$\begin{aligned} \text{Perimeter of slab at } 1.5d \text{ (} d = 170 \text{ mm)} \\ &= 4 \times (2000 + 3 \times 170) = 10040 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Shear stress, } v &= \frac{420 \times 10^3}{10040 \times 170} \\ &= 0.25 \text{ N/mm}^2 < v_c \quad \text{for minimum percentage of reinforcement} \end{aligned}$$

No shear reinforcement is required at internal columns of floor slab.

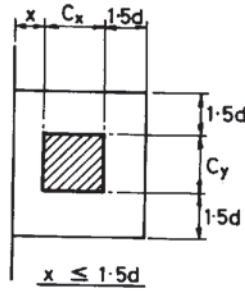
Similarly check for an external column and a corner column.

Rules for calculation of perimeters of external and corner columns



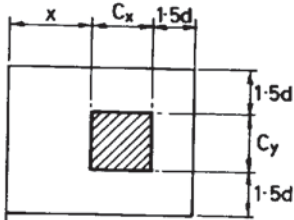
$$P = 2C_x + C_y + 6d$$

EDGE COLUMN ON
EDGE OF SLAB



$$P = 2(x + C_x) + C_y + 6d$$

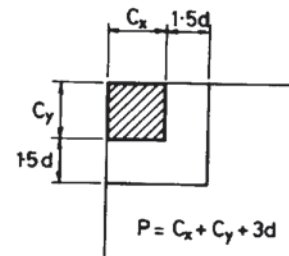
EDGE COLUMN INSIDE
SLAB



$$x > 1.5d$$

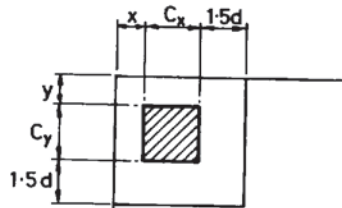
P = lesser of :-
 $2(x + C_x) + C_y + 6d$
OR $2(C_x + C_y) + 12d$

EDGE COLUMN INSIDE
SLAB



$$P = C_x + C_y + 3d$$

CORNER COLUMN ON
ON EDGE OF SLAB

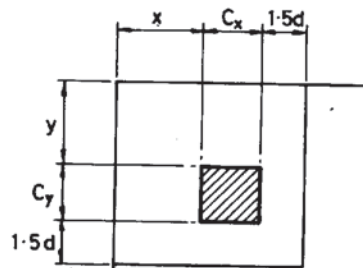


$$x \text{ and } y \leq 1.5d$$

$$P = C_x + C_y + x + y + 3d$$

CORNER COLUMN INSIDE
SLAB

d = Average effective
depth of slab



$$x \text{ and } y > 1.5d$$

P = lesser of :-
 $C_x + C_y + x + y + 3d$
OR $2(C_x + C_y) + 12d$

CORNER COLUMN INSIDE
SLAB

SK 9/39 Punching shear perimeters for flat slab.

The illustrations show the different column configurations with respect to a free edge and the corresponding perimeters for the calculation of punching shear stresses.

When the column face is more than $1.5d$ away from a free edge of slab, then there are two alternative perimeters possible as illustrated. Take the least of these two alternatives for the calculation of punching shear stress.

Punching shear at roof slab

Check joint 3 – edge column.

$$V_t = 130 \text{ kN}$$

The frame action considered is in the $x-x$ direction as explained in Section 9.3.5.

$$V_{\text{eff}} = 1.25V_t = 1.25 \times 130 = 162.5 \text{ kN}$$

$$d = 170 \text{ mm assumed}$$

$$1.5d = 1.5 \times 170 = 255 \text{ mm}$$

Shear stress at column perimeter ($U_o = 3 \times 400 = 1200$)

$$= \frac{V_{\text{eff}}}{U_o d} = \frac{162.5 \times 10^3}{1200 \times 170} = 0.8 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \quad \text{OK}$$

$$\text{Shear stress at } 1.5d = v = \frac{V_{\text{eff}}}{Ud}$$

$$U = [2 \times (400 + 255)] + (400 + 510) \\ = 2220$$

$$\therefore v = \frac{162.5 \times 10^3}{2220 \times 170} \\ = 0.43 \text{ N/mm}^2$$

Assume minimum percentage of tensile reinforcement in slab.

$$v_c = 0.48 \text{ N/mm}^2 \text{ for Grade 40 concrete and an effective depth of } 170 \text{ mm.}$$

No shear reinforcement is necessary.

Note: The punching shear check should also be carried out for the flat slab spanning in the long direction and the worst result should be used.

Step 11 Check span/effective depth ratio

Follow Step 11 of Section 3.3

Step 12 Curtailment of bars

Follow Step 12 of Section 3.3.

Step 13 Spacing of bars

Follow Step 13 of Section 3.3.

Step 14 Check early thermal cracking

Follow Step 14 of Section 3.3.

Step 15 Calculate minimum reinforcement

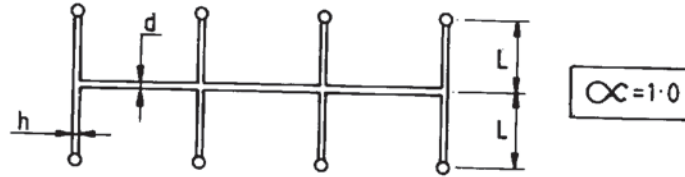
Follow Step 9 and Step 15 of Section 3.3.

Step 16 Calculate flexural crack width
Follow Step 16 of Section 3.3.

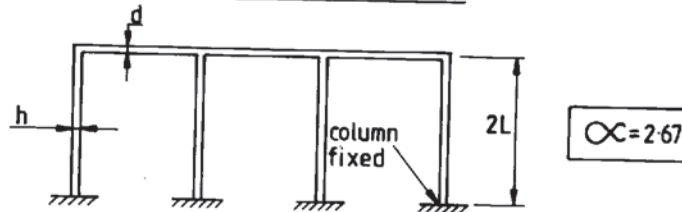
Step 17 Design of connections
Follow Chapter 11

9.6 TABLES AND GRAPHS FOR CHAPTER 9

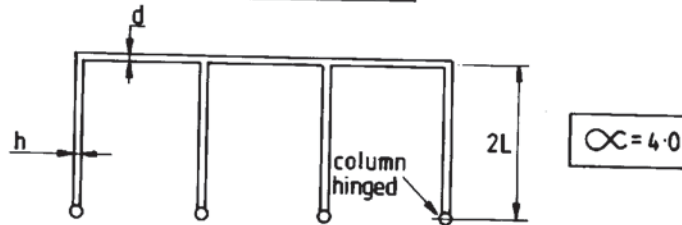
How to use Tables 9.1 to 9.6



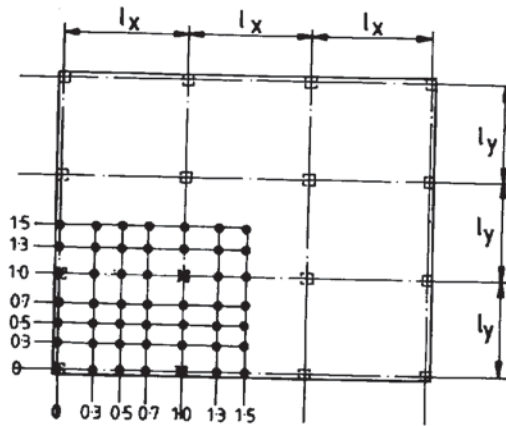
Intermediate floor. Typical section



First floor. Typical section

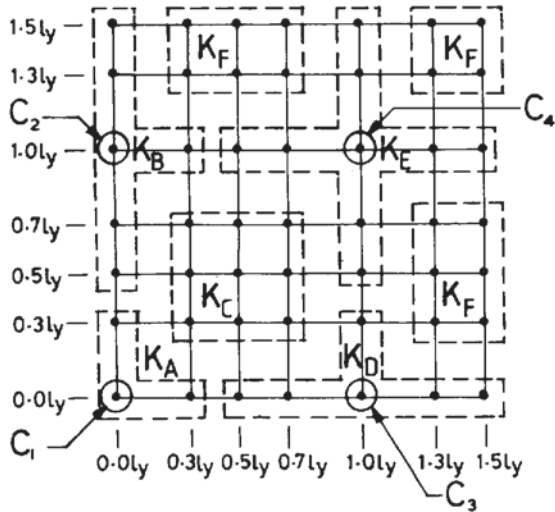


First floor. Typical section



Plan showing points for which coefficients are in tables 9.1 to 9.6

SK 9/40 Sketches to be used in conjunction with Tables 9.1–9.6.

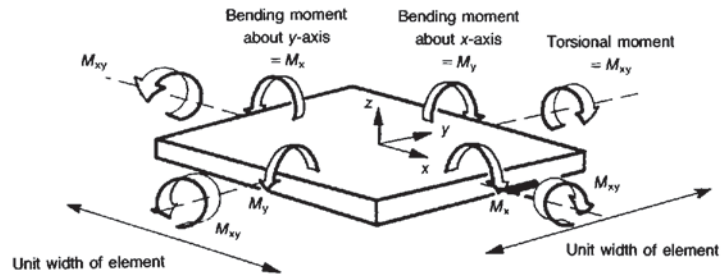


SK 9/41 Zones of stiffness correction factors to be applied to points of interest.

Points to note:

- (1) The flat slab system should have at least 3 spans in the l_x direction and 3 spans in the l_y direction.
- (2) The coefficients are valid for equal spans in the l_x and l_y direction. They may be used up to a maximum variation of 20% in the span lengths.
- (3) The tables can be used only for uniformly distributed loads with all spans loaded simultaneously with the maximum load.
- (4) To account for the possible increase in moment due to variation of live loads in different panels of the flat slab, no redistribution of moments should be carried out.
- (5) For horizontal loading a separate frame analysis should be carried out and the appropriate moments will be combined with the vertical load moments. In general, flat slab construction should be fully braced and the horizontal load should be carried entirely by a shear-wall system.
- (6) The coefficients are applicable to a corner panel, an edge panel with a free edge in the l_x direction, an edge panel with a free edge in the l_y direction and an internal panel.
- (7) The moment triad (M_x , M_y and M_{xy}) obtained by this method of analysis should be combined using the Wood–Armer method as described in Section 1.12.

Sign convention for bending moments (+ve in directions shown)



Note: A positive moment denotes hogging. This sign convention is opposite to the Wood–Armer convention. Reverse the signs of the moments before carrying out Wood–Armer combination as per Section 1.12.

Step-by-step analysis procedure

Step 1: Determine value of α (see SK 9/40 and find l_x/l_y).

Step 2: Assume d is the thickness of slab and h is the dimension of a side of a square column.

Step 3: From available L , as shown in SK 9/40, determine $S = \alpha d^3 L/h^4$.

Step 4: Select a point of interest from SK 9/40 or SK 9/41 where the moments have to be found. Corresponding to the zone of influence, find appropriate stiffness connection factor K depending on S from Graphs 9.1 to 9.18.

Step 5: Find moment coefficients from Tables 9.1 to 9.3 corresponding to l_x/l_y and the location of the point of interest.

Step 6: Find the ultimate uniformly distributed load on the flat slab = n kN/m².

Step 7: Find moment triads:

$$M_x = nC_x K_x l_y^2 \text{ kNm/m}$$

$$M_y = nC_y K_y l_x^2 \text{ kNm/m}$$

$$M_{xy} = nC_{xy} K_{xy} l_x l_y \text{ kNm/m}$$

Step 8: Carry out Wood–Armer combination.

Step 9: Find column reactions corresponding to l_x/l_y (see SK 9/41 for column locations).

Step 10: The moments obtained using these coefficients are in kNm/m. Find the effective width b_e as in Section 9.3.4. Multiply the moment obtained by analysis at edge and corner columns with the effective width b_e to find the slab-to-column connection moment. This transfer moment should be less than $M_{t,\max}$ as defined in Section 9.3.4.

Reaction at column $C_1 = nC_1 l_x l_y$

Reaction at column $C_2 = nC_2 l_x l_y$, etc.

Note: If in the zone of K_A (as shown in SK 9/41) the point of interest is located, then to find M_x use stiffness correction factor K_{AX} corresponding to S and l_x/l_y as in Graph 9.1. Similarly if the point of interest lies in the zone of K_F then to find M_x use K_{FX} as in Graph 9.16.

The benefits of using these tables and graphs are that the analysis can be done very quickly and the necessity of carrying out the two analyses for the two orthogonal directions may be avoided. These tables can also be used for the analyses of raft foundation where the loading n may be assumed to be uniformly distributed over an inverted flat slab. The total loads from a structure will be assumed uniformly distributed at the underside of the raft.

Step 11: Calculate total column moments using Table 9.7 and divide the total moment between the columns at the junction depending on their relative stiffness. The stiffer the column, the more moment it will carry. The stiffness of a column may be calculated as I/l where I is the moment of inertia and l is the effective height.

Table 9.1 Bending moment coefficients for design of flat slabs ($I_x/I_y = 1.0$).

Location of point of interest	$0I_x$	$0.3I_x$	$0.5I_x$	$0.7I_x$	$1.0I_x$	$1.3I_x$	$1.5I_x$	Moment coefficient, C
$1.5I_y$	-0.011 84	-0.053 70	-0.056 97	-0.024 96	+0.026 33	-0.005 54	-0.022 30	C_x
	-0.049 87	-0.033 00	-0.027 94	-0.033 01	-0.043 58	-0.028 44	-0.022 30	C_y
	0.005 41	0.002 96	0.001 00	0.004 00	0.001 59	0.003 80	0.001 13	C_{xy}
$1.3I_y$	-0.010 48	-0.057 39	-0.062 49	-0.026 20	+0.046 90	-0.007 57	-0.028 44	C_x
	-0.027 76	-0.016 67	-0.013 22	-0.012 65	-0.019 54	-0.007 57	-0.005 54	C_y
	0.020 30	0.011 03	0.003 38	0.014 76	0.007 44	0.014 20	0.003 80	C_{xy}
$1.0I_y$	+0.054 82	-0.070 14	-0.075 60	-0.036 47	+0.156 56	-0.019 54	-0.043 58	C_x
	+0.115 71	+0.027 04	+0.015 21	-0.042 47	+0.156 56	+0.046 90	+0.026 33	C_y
	0.031 89	0.006 19	0.001 59	0.008 19	0.042 91	0.007 44	0.001 59	C_{xy}
$0.7I_y$	-0.010 49	-0.057 49	-0.063 57	-0.029 41	+0.042 47	-0.012 65	-0.033 01	C_x
	-0.041 32	-0.032 18	-0.029 93	-0.029 41	-0.036 47	-0.026 20	-0.024 96	C_y
	0.025 24	0.012 14	0.003 95	0.017 90	0.008 19	0.014 76	0.004 00	C_{xy}
$0.5I_y$	-0.001 57	-0.052 45	-0.057 18	-0.029 93	+0.015 21	-0.013 22	-0.027 94	C_x
	-0.076 03	-0.060 63	-0.057 18	-0.063 57	-0.075 60	-0.062 49	-0.056 97	C_y
	0.005 62	0.002 62	0.001 13	0.003 95	0.001 59	0.003 38	0.001 00	C_{xy}
$0.3I_y$	-0.010 59	-0.054 23	-0.060 63	-0.032 18	+0.027 04	-0.016 67	-0.033 00	C_x
	-0.065 15	-0.054 23	-0.052 45	-0.057 49	-0.070 14	-0.057 39	-0.053 70	C_y
	0.014 19	0.007 44	0.002 62	0.012 14	0.006 19	0.011 03	0.002 96	C_{xy}
$0I_y$	+0.063 70	-0.065 15	-0.076 03	-0.041 32	+0.115 71	-0.027 76	-0.049 87	C_x
	+0.063 70	-0.010 59	-0.011 57	-0.010 49	+0.054 82	-0.010 48	-0.011 84	C_y
	0.012 77	0.014 19	0.005 62	0.025 24	0.031 89	0.020 30	0.005 41	C_{xy}

$C_1 = 0.194355$ $C_2 = 0.45153$ $C_3 = 0.45153$ $C_4 = 1.15263$

Table 9.2 Bending moment coefficients for design of flat slabs ($l_x/l_y = 1.2$).

Location of point of interest	$0l_x$	$0.3l_x$	$0.5l_x$	$0.7l_x$	$1.0l_x$	$1.3l_x$	$1.5l_x$	Moment coefficient, C
$1.5l_y$	-0.01657	-0.07980	-0.08586	-0.03473	+0.05340	-0.00898	-0.03784	C_x
	-0.05758	-0.03557	-0.02828	-0.03295	-0.04754	-0.02534	-0.01780	C_y
	0.00608	0.00347	0.00108	0.00474	0.00236	0.00466	0.00123	C_{xy}
$1.3l_y$	-0.01414	-0.08348	-0.09097	-0.03764	-0.07768	-0.01256	-0.04371	C_x
	-0.03235	-0.02082	-0.01716	-0.01332	-0.01930	-0.00593	-0.00543	C_y
	0.02211	0.01211	0.00337	0.01624	0.01089	0.01604	0.00383	C_{xy}
$1.0l_y$	+0.07262	-0.09512	-0.10176	-0.05003	+0.19583	-0.02621	-0.05675	C_x
	+0.12706	+0.01542	+0.00164	-0.03408	+0.18274	+0.03907	+0.01480	C_y
	0.04229	0.00573	0.00158	0.00776	0.05087	0.00707	0.00145	C_{xy}
$0.7l_y$	-0.01399	-0.08230	-0.09065	-0.04080	+0.07160	-0.01733	-0.04690	C_x
	-0.04465	-0.03575	-0.03378	-0.03032	-0.03656	-0.02569	-0.02623	C_y
	0.02831	0.01360	0.00421	0.02085	0.01213	0.01747	0.00432	C_{xy}
$0.5l_y$	-0.01604	-0.07664	-0.08390	-0.03994	+0.03919	-0.01588	-0.04109	C_x
	-0.08182	-0.06117	-0.05595	-0.05809	-0.08095	-0.06046	-0.05311	C_y
	0.00710	0.00316	0.00148	0.00502	0.00236	0.00431	0.00115	C_{xy}
$0.3l_y$	-0.01365	-0.07796	-0.08693	-0.04249	+0.05316	-0.02017	-0.04558	C_x
	-0.06907	-0.05432	-0.05111	-0.05730	-0.07482	-0.05576	-0.05069	C_y
	0.01457	0.00963	0.00299	0.01417	0.00935	0.01330	0.00331	C_{xy}
$0l_y$	+0.09004	-0.08934	-0.10261	-0.05444	+0.15200	-0.03380	-0.06265	C_x
	+0.06476	-0.01134	-0.01169	-0.01106	+0.05945	-0.01081	-0.01176	C_y
	0.01871	0.01902	0.00641	0.03097	0.03657	0.02540	0.00656	C_{xy}

$C_1 = 0.195075$ $C_2 = 0.44811$ $C_3 = 0.453375$ $C_4 = 1.15344$

Table 9.3 Bending moment coefficients for design of flat slabs ($I_x/I_y = 1.4$).

Location of point of interest	$0l_x$	$0.3l_x$	$0.5l_x$	$0.7l_x$	$1.0l_x$	$1.3l_x$	$1.5l_x$	Moment coefficient, C
$1.5l_y$	-0.02136	-0.10961	-0.11941	-0.04705	+0.08741	-0.01469	-0.05713	C_x
	-0.06491	-0.03952	-0.03062	-0.03282	-0.05018	-0.02241	-0.01520	C_y
	0.00639	0.00369	0.00100	0.00498	0.00320	0.01037	0.00113	C_{xy}
$1.3l_y$	-0.01752	-0.11318	-0.12362	-0.05114	+0.11466	-0.01923	-0.06200	C_x
	-0.03625	-0.02623	-0.02280	-0.01445	-0.01742	-0.00486	-0.00687	C_y
	0.02248	0.01222	0.00296	0.01615	0.01479	0.01609	0.00335	C_{xy}
$1.0l_y$	+0.09384	-0.12354	-0.13173	-0.06459	+0.23834	-0.03332	-0.07210	C_x
	+0.13729	+0.00397	-0.01129	+0.02551	+0.20889	+0.03142	+0.00431	C_y
	0.05628	0.00527	0.00170	0.00717	0.05755	0.00649	0.00125	C_{xy}
$0.7l_y$	-0.01723	-0.11078	-0.12186	-0.05410	+0.10672	-0.02342	-0.06343	C_x
	-0.04721	-0.03997	-0.03854	-0.03142	-0.03513	-0.02532	-0.02823	C_y
	0.03003	0.01418	0.00411	0.02227	0.01662	0.01867	0.00422	C_{xy}
$0.5l_y$	-0.02056	-0.10476	-0.11551	-0.05103	+0.07003	-0.02048	-0.05780	C_x
	-0.08747	-0.06227	-0.05539	-0.06281	-0.08560	-0.05789	-0.04970	C_y
	0.00842	0.00355	0.00170	0.00575	0.00318	0.00483	0.00116	C_{xy}
$0.3l_y$	-0.01606	-0.10580	-0.11820	-0.05479	+0.08586	-0.02520	-0.06155	C_x
	-0.07295	-0.05462	-0.04976	-0.05667	-0.07905	-0.05361	-0.04740	C_y
	0.01377	0.01071	0.00382	0.01612	0.01305	0.01473	0.00343	C_{xy}
$0l_y$	+0.12167	-0.11753	-0.13360	-0.06868	+0.19234	-0.04028	-0.07779	C_x
	+0.06593	-0.01189	-0.01158	-0.01135	+0.06438	-0.01080	-0.01135	C_y
	0.03124	0.02358	0.00741	0.03596	0.04237	0.02947	0.00739	C_{xy}

$C_1 = 0.19683$ $C_2 = 0.445545$ $C_3 = 0.45477$ $C_4 = 1.1529$

Table 9.4 Bending moment coefficients for design of flat slabs ($I_x/I_y = 1.6$).

Location of point of interest	$0l_x$	$0.3l_x$	$0.5l_x$	$0.7l_x$	$1.0l_x$	$1.3l_x$	$1.5l_x$	Moment coefficient, C
$1.5l_y$	-0.02596	-0.14272	-0.15690	-0.06172	+0.12785	-0.02244	-0.07938	C_x
	-0.07131	-0.04461	-0.03496	-0.03274	-0.05106	-0.01984	-0.01475	C_y
	0.00638	0.00367	0.00083	0.00484	0.00411	0.00478	0.00091	C_{xy}
$1.3l_y$	-0.02028	-0.14612	-0.16010	-0.06652	+0.15746	-0.02746	-0.08306	C_x
	-0.03924	-0.03265	-0.02981	-0.01601	-0.01375	-0.00439	-0.00969	C_y
	0.02163	0.01167	0.00236	0.01496	0.01912	0.01489	0.00261	C_{xy}
$1.0l_y$	+0.11858	-0.15531	-0.16570	-0.08020	+0.28462	-0.04116	-0.09017	C_x
	+0.14626	-0.00727	-0.02353	+0.01704	+0.23486	+0.02416	-0.00511	C_y
	0.07490	0.00493	0.00211	0.00649	0.06314	0.00578	0.00102	C_{xy}
$0.7l_y$	-0.01988	-0.14273	-0.15713	-0.06914	+0.14768	-0.03091	-0.08271	C_x
	-0.04901	-0.04471	-0.04392	-0.03274	-0.03231	-0.02516	-0.03077	C_y
	0.03044	0.01397	0.00373	0.02243	0.02162	0.01871	0.00386	C_{xy}
$0.5l_y$	-0.02479	-0.13655	-0.15162	-0.06494	+0.10749	-0.02697	-0.07773	C_x
	-0.09266	-0.06388	-0.05566	-0.06185	-0.08934	-0.05511	-0.04713	C_y
	0.00954	0.00383	0.00182	0.00602	0.00408	0.00504	0.00110	C_{xy}
$0.3l_y$	-0.01737	-0.13751	-0.15419	-0.06902	+0.12493	-0.03182	-0.08087	C_x
	-0.07649	-0.05520	-0.04868	-0.05576	-0.08263	-0.05125	-0.04433	C_y
	0.01189	0.01251	0.00267	0.01759	0.01727	0.01559	0.00346	C_{xy}
$0l_y$	+0.15859	-0.14962	-0.16914	-0.08415	+0.23712	-0.04757	-0.09590	C_x
	+0.06699	-0.01231	-0.01139	-0.01145	+0.06950	-0.01061	-0.01082	C_y
	0.05131	0.02810	0.00830	0.04046	0.04891	0.03267	0.00799	C_{xy}

$C_1 = 0.199125$ $C_2 = 0.443385$ $C_3 = 0.45576$ $C_4 = 1.15173$

Table 9.5 Bending moment coefficients for design of flat slabs ($I_x/I_y = 1.8$).

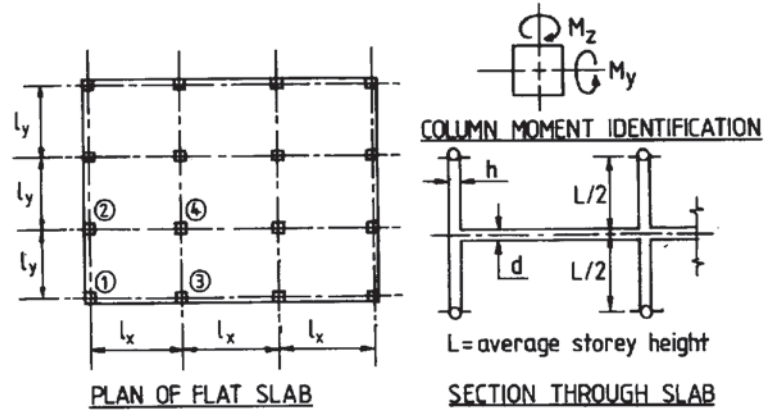
Location of point of interest	$0I_x$	$0.3I_x$	$0.5I_x$	$0.7I_x$	$1.0I_x$	$1.3I_x$	$1.5I_x$	Moment coefficient, C
1.5I _y	-0.03010	-0.17890	-0.19796	-0.07854	+0.17436	-0.03203	-0.10420	C _x
	-0.07652	-0.05063	-0.04105	-0.03279	-0.05001	-0.01774	-0.01633	C _y
	0.00608	0.00350	0.00063	0.00442	0.00511	0.00431	0.00066	C _{xy}
1.3I _y	-0.02210	-0.18209	-0.20023	-0.08364	+0.20582	-0.03750	-0.10677	C _x
	-0.04120	-0.03986	-0.03792	-0.01791	-0.00829	-0.00447	-0.01368	C _y
	0.01976	0.01078	0.00171	0.01311	0.02388	0.01293	0.00181	C _{xy}
1.0I _y	+0.14687	-0.19028	-0.20370	-0.09695	+0.33505	-0.04997	-0.11126	C _x
	+0.15402	-0.01830	-0.03510	+0.00888	+0.26064	+0.01736	-0.01359	C _y
	0.09911	0.00489	0.00247	0.00631	0.06783	0.00498	0.00081	C _{xy}
0.7I _y	-0.02183	-0.17800	-0.19642	-0.08582	+0.19437	-0.03982	-0.10488	C _x
	-0.05007	-0.04986	-0.04970	-0.03425	-0.02818	-0.02523	-0.03368	C _y
	0.02965	0.01313	0.00344	0.02159	0.02708	0.01792	0.00337	C _{xy}
0.5I _y	-0.02840	-0.17182	-0.19196	-0.08093	+0.15136	-0.03531	-0.10074	C _x
	-0.09718	-0.06594	-0.05675	-0.06072	-0.09204	-0.05232	-0.04552	C _y
	0.01047	0.00403	0.00184	0.00589	0.00516	0.00503	0.00099	C _{xy}
0.3I _y	-0.01716	-0.17285	-0.19464	-0.08511	+0.17018	-0.04005	-0.10347	C _x
	-0.07944	-0.05607	-0.04804	-0.54680	-0.08544	-0.04887	-0.04177	C _y
	0.00908	0.01458	0.00396	0.01906	0.02196	0.01615	0.00350	C _{xy}
0I _y	+0.20072	-0.18539	-0.20919	-0.10093	+0.28659	-0.05594	-0.11731	C _x
	+0.06785	-0.01273	-0.01121	-0.01143	+0.07473	-0.01029	-0.01027	C _y
	0.08002	0.03292	0.00914	0.04472	0.05597	0.03522	0.00844	C _{xy}

C₁ = 0.20178 C₂ = 0.441495 C₃ = 0.45639 C₄ = 1.15038

Table 9.6 Bending moment coefficients for design of flat slabs ($l_x/l_y = 2.0$).

Location of point of interest	$0l_x$	$0.3l_x$	$0.5l_x$	$0.7l_x$	$1.0l_x$	$1.3l_x$	$1.5l_x$	Moment coefficient, C
$1.5l_y$	-0.03352	-0.21798	-0.24238	-0.09727	+0.22671	-0.04331	-0.13147	C_x
	-0.08045	-0.05733	-0.04855	-0.03300	-0.04700	-0.01614	-0.01960	C_y
	0.00553	0.00326	0.00044	0.00386	0.00621	0.00368	0.00041	C_{xy}
$1.3l_y$	-0.02266	-0.22093	-0.24390	-0.10237	+0.25960	-0.04825	-0.13312	C_x
	-0.04210	-0.04764	-0.04682	-0.02007	-0.00105	-0.00502	-0.01861	C_y
	0.01707	0.00981	0.00113	0.01098	0.02901	0.01064	0.00107	C_{xy}
$1.0l_y$	+0.17869	-0.22830	-0.24572	-0.11492	+0.38991	-0.05993	-0.13551	C_x
	+0.16068	-0.02909	-0.04607	+0.00116	+0.28634	-0.01105	-0.02132	C_y
	0.12976	0.00537	0.00275	0.00633	0.07181	0.00436	0.00102	C_{xy}
$0.7l_y$	-0.02187	-0.21636	-0.23968	-0.10405	+0.24672	-0.05017	-0.13005	C_x
	-0.05039	-0.05533	-0.05573	-0.03591	-0.02278	-0.02554	-0.03688	C_y
	0.02777	0.01179	0.00331	0.02000	0.03292	0.01656	0.00283	C_{xy}
$0.5l_y$	-0.03105	-0.21043	-0.23636	-0.09883	+0.20147	-0.04540	-0.12677	C_x
	-0.10089	-0.06837	-0.05857	-0.05954	-0.09363	-0.04968	-0.04482	C_y
	0.01121	0.00418	0.00178	0.00588	0.00635	0.00488	0.00090	C_{xy}
$0.3l_y$	-0.01505	-0.21163	-0.23936	-0.10299	+0.22144	-0.04992	-0.12932	C_x
	-0.08166	-0.05719	-0.04790	-0.05350	-0.08741	-0.04657	-0.03984	C_y
	0.00549	0.01703	0.00446	0.02071	0.02705	0.01663	0.00361	C_{xy}
$0l_y$	+0.24789	-0.22465	-0.25366	-0.11907	+0.34090	-0.06557	-0.14215	C_x
	+0.06845	-0.01316	-0.01109	-0.01131	+0.08006	-0.00991	-0.00979	C_y
	0.11841	0.03847	0.01002	0.04900	0.06341	0.03730	0.00882	C_{xy}

 $C_1 = 0.204615$ $C_2 = 0.43974$ $C_3 = 0.45666$ $C_4 = 1.14903$



SK 9/42 Column number identification.

Table 9.7 Bending moment coefficient for design of columns in flat slab construction.

l_x/l_y	1	1.2	1.4	1.6	1.8	2	Moment coefficients
Column no.							
1	0.011 75	0.013 45	0.015 26	0.017 17	0.019 16	0.021 23	C_{cy}
1	-0.011 75	-0.017 92	-0.025 94	-0.036 03	-0.048 36	-0.063 07	C_{cz}
2	-0.002 44	-0.002 33	-0.002 24	-0.002 18	-0.002 16	-0.002 18	C_{cy}
2	-0.022 80	-0.032 71	-0.045 25	-0.060 73	-0.079 39	-0.101 47	C_{cz}
3	0.022 80	0.027 65	0.033 01	-0.038 85	0.045 13	0.051 84	C_{cy}
3	0.002 44	0.004 41	0.007 06	0.010 40	0.014 45	0.019 19	C_{cz}
4	-0.003 76	-0.003 93	-0.004 18	-0.004 54	-0.005 06	-0.005 71	C_{cy}
4	0.003 76	0.006 59	0.010 28	0.014 84	0.020 31	0.026 67	C_{cz}

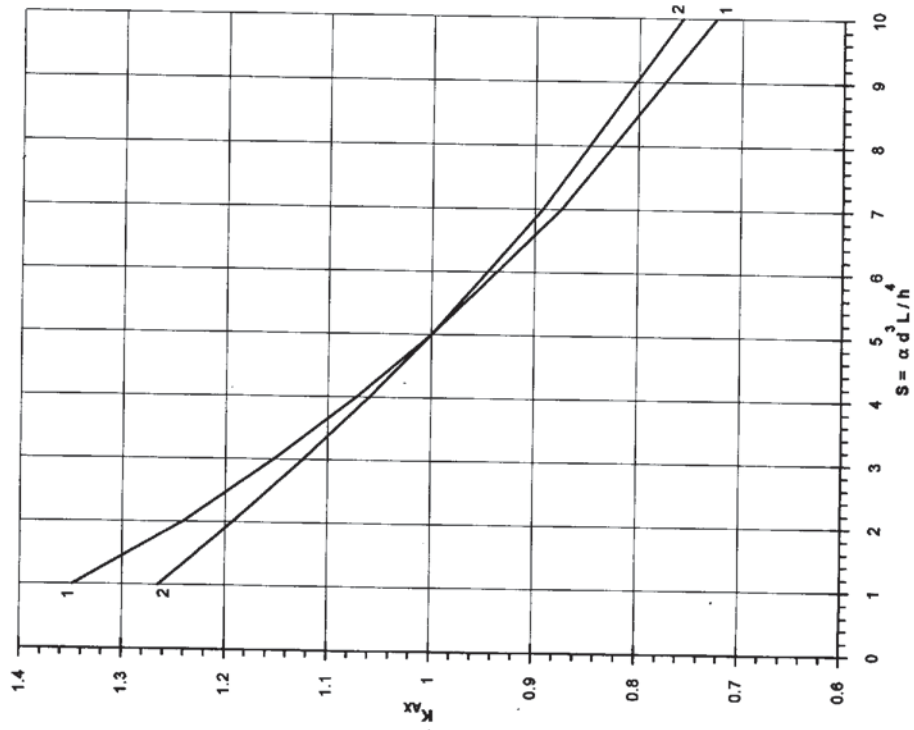
$$\text{Column total moment} = C_c \times R \times l_y^3 \times \eta$$

where η = load per unit area on slab.

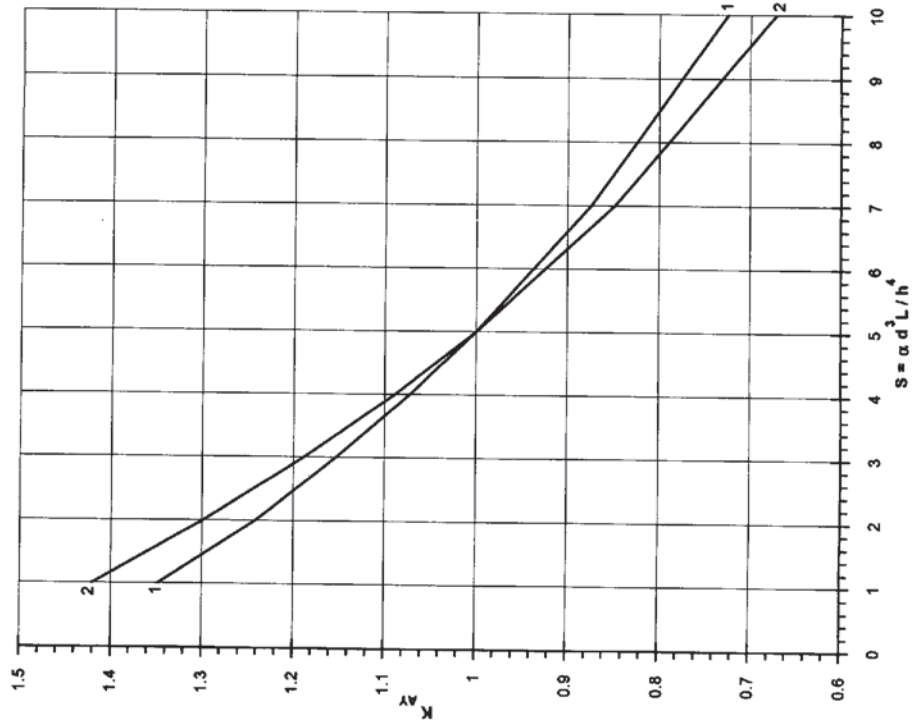
See Graphs 9.19–9.26 for factors R_y and R_z .

Note: Divide total moment in column to top and bottom column in proportion to their stiffness.

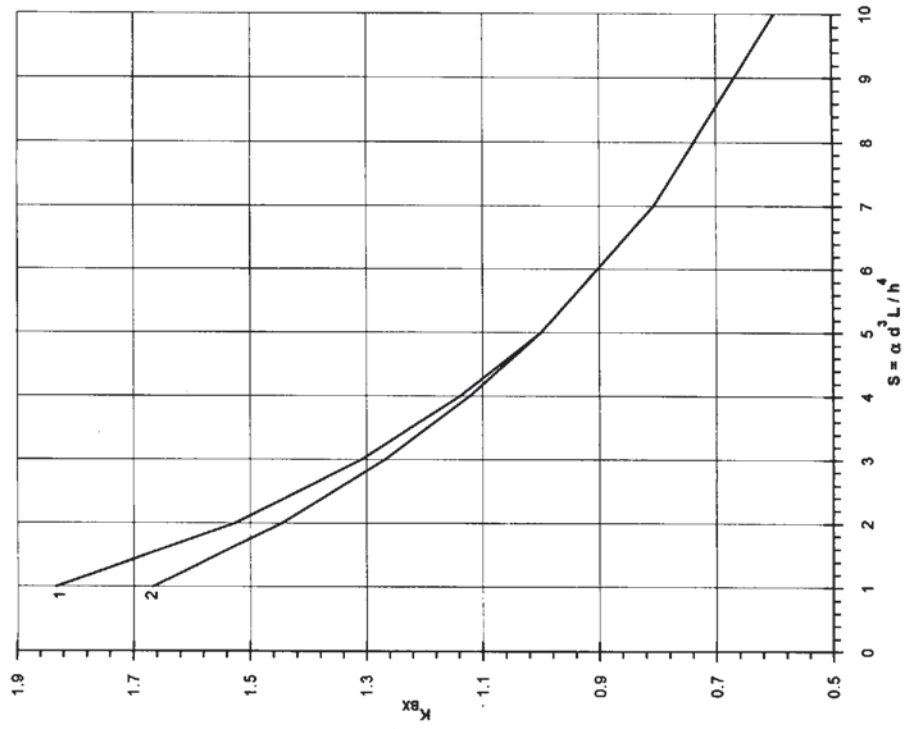
Graph 9.1 K_{Ax} for M_x . Stiffness correction coefficients for Zone A, curves for $I_x/I_y = 1$ and 2.



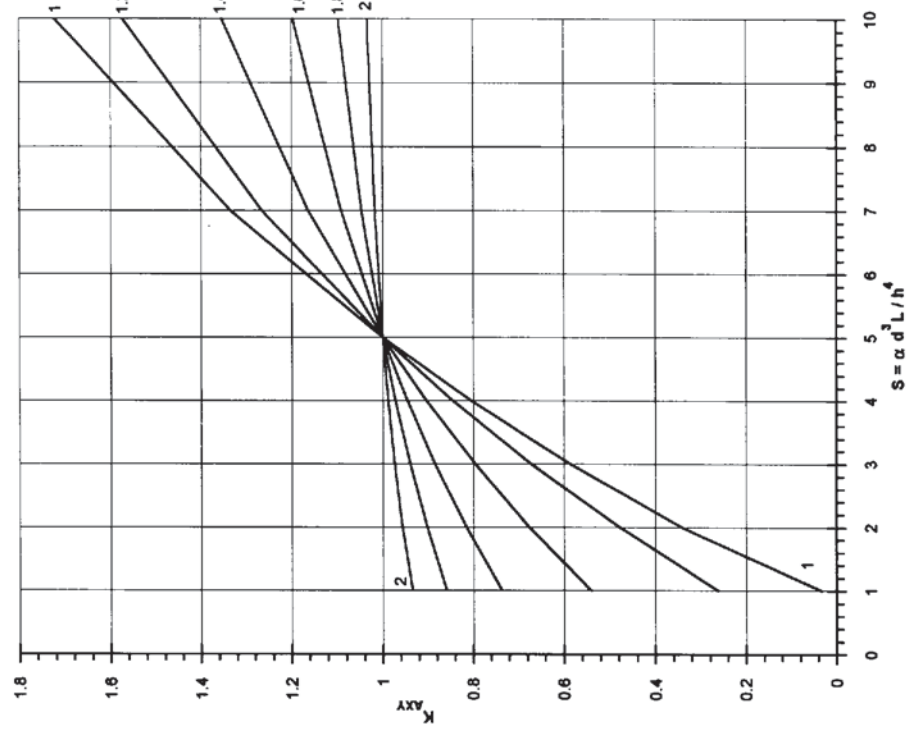
Graph 9.2 K_{Ay} for M_y . Stiffness correction coefficients for Zone A, curves for $I_x/I_y = 1$ and 2.



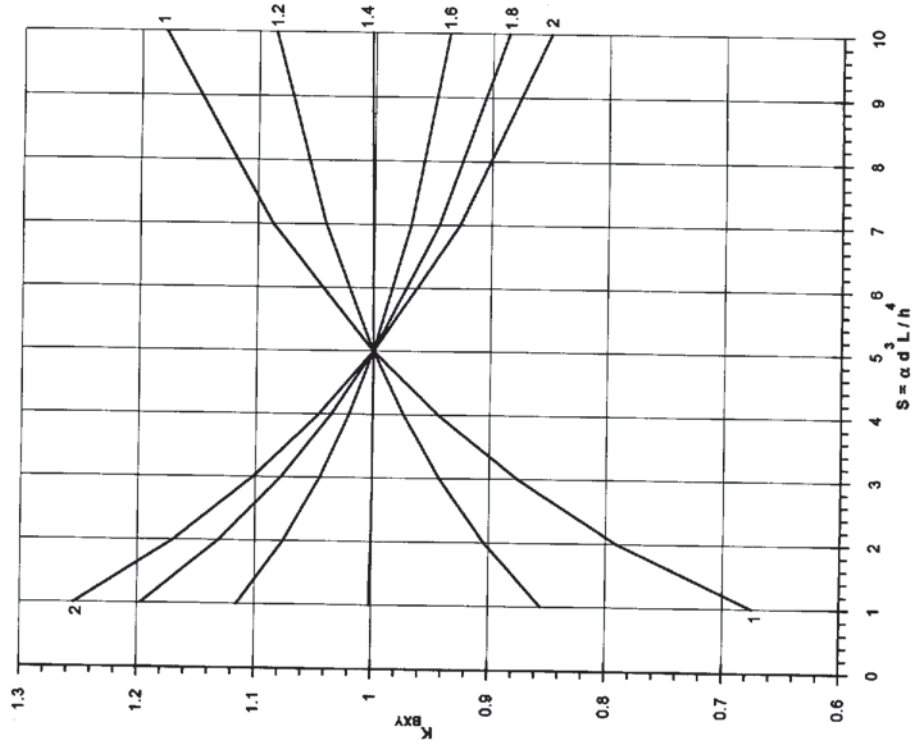
Graph 9.4 K_{Bx} for M_x . Stiffness correction coefficients for Zone B, curves for $I_x/I_y = 1$ and 2.



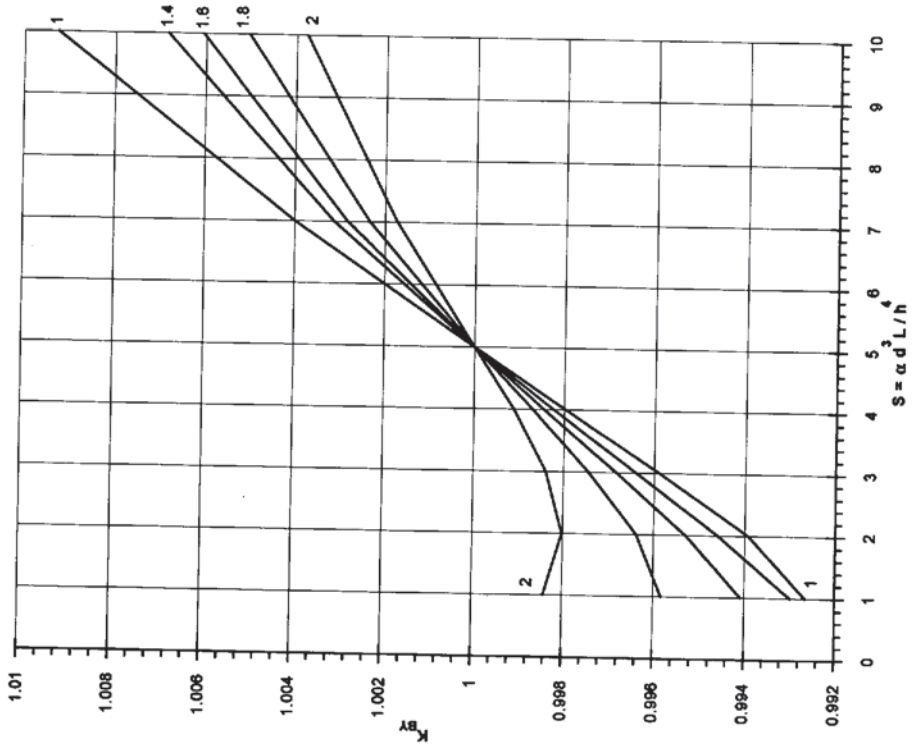
Graph 9.3 K_{Axy} for M_{xy} . Stiffness correction coefficients for Zone A, curves for $I_x/I_y = 1, 1.2, 1.4, 1.6, 1.8$ and 2.



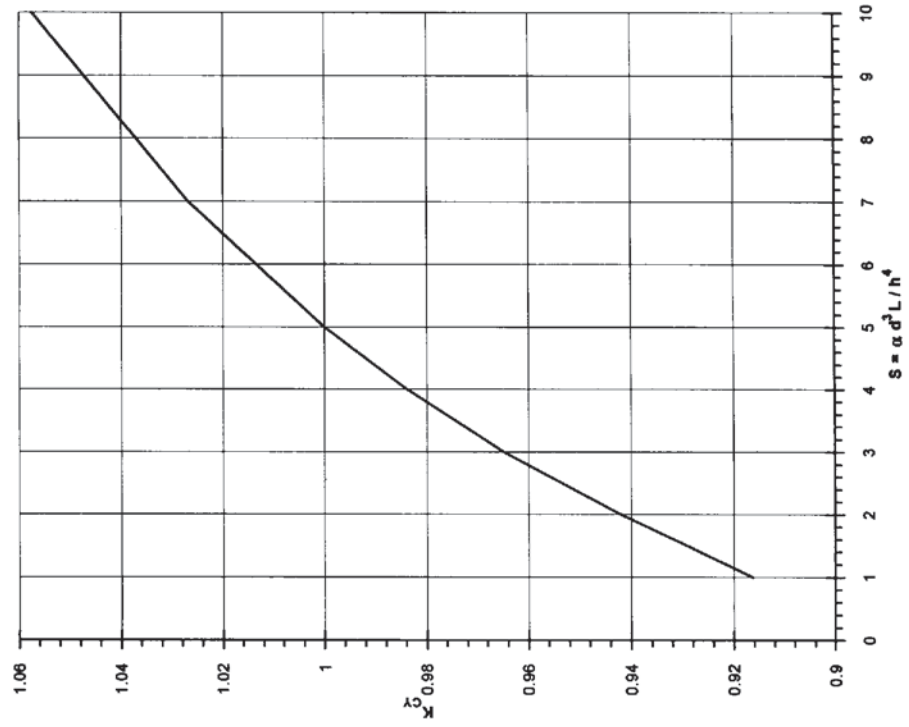
Graph 9.6 K_{Bxy} for M_{xy} . Stiffness correction coefficients for Zone B, curves for $l_x/l_y = 1, 1.2, 1.4, 1.6, 1.8$ and 2.



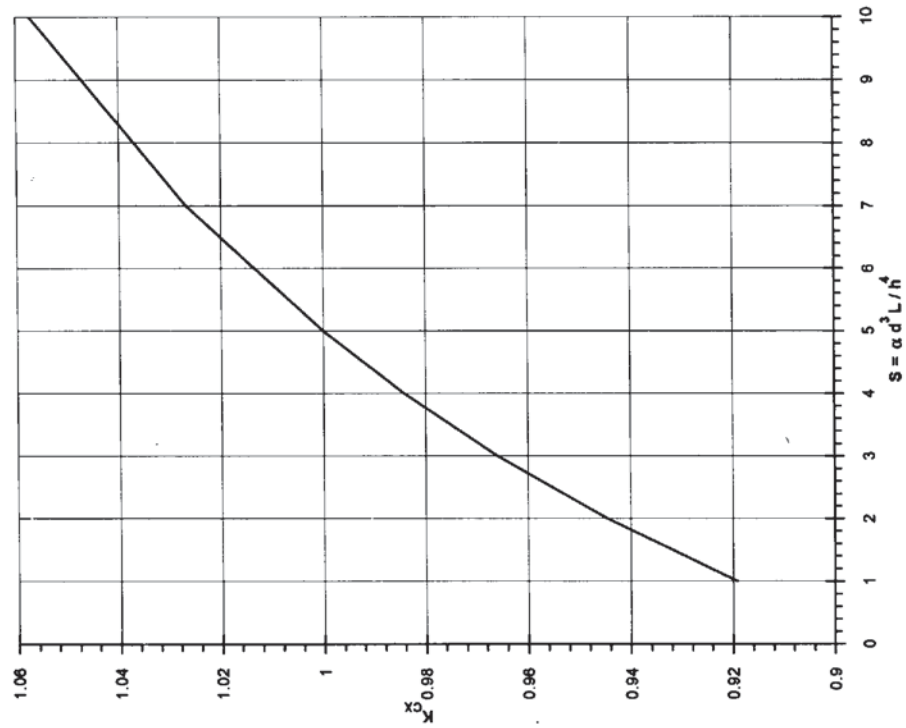
Graph 9.5 K_{By} for M_y . Stiffness correction coefficients for Zone B, curves for $l_x/l_y = 1, 1.4, 1.6, 1.8$ and 2.



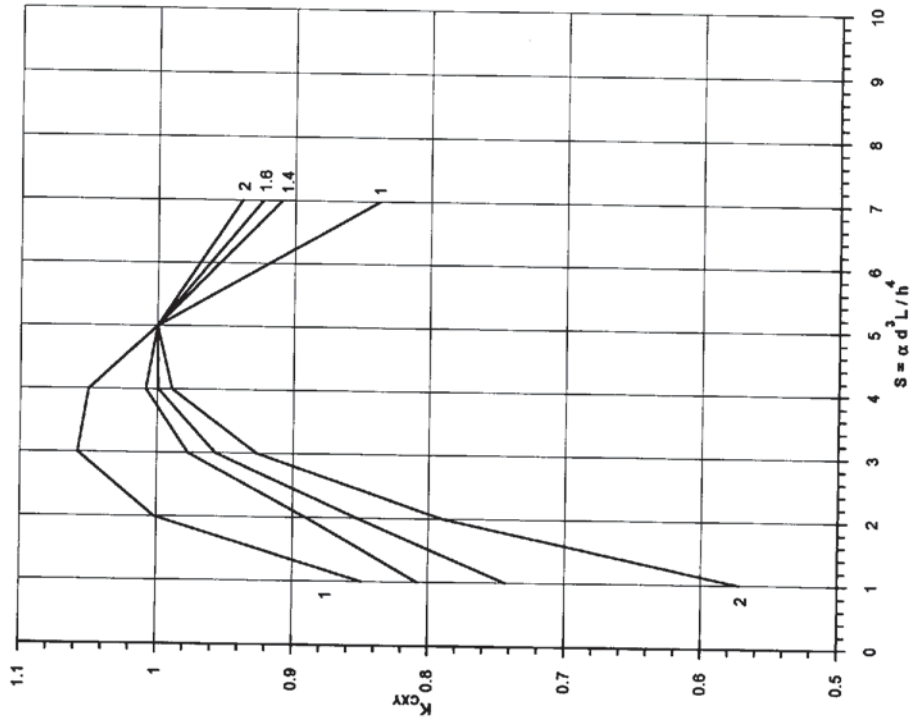
Graph 9.8 K_{Cy} for M_y . Stiffness correction coefficients for Zone C, curves for $I_x/I_y = 1$ and 2.



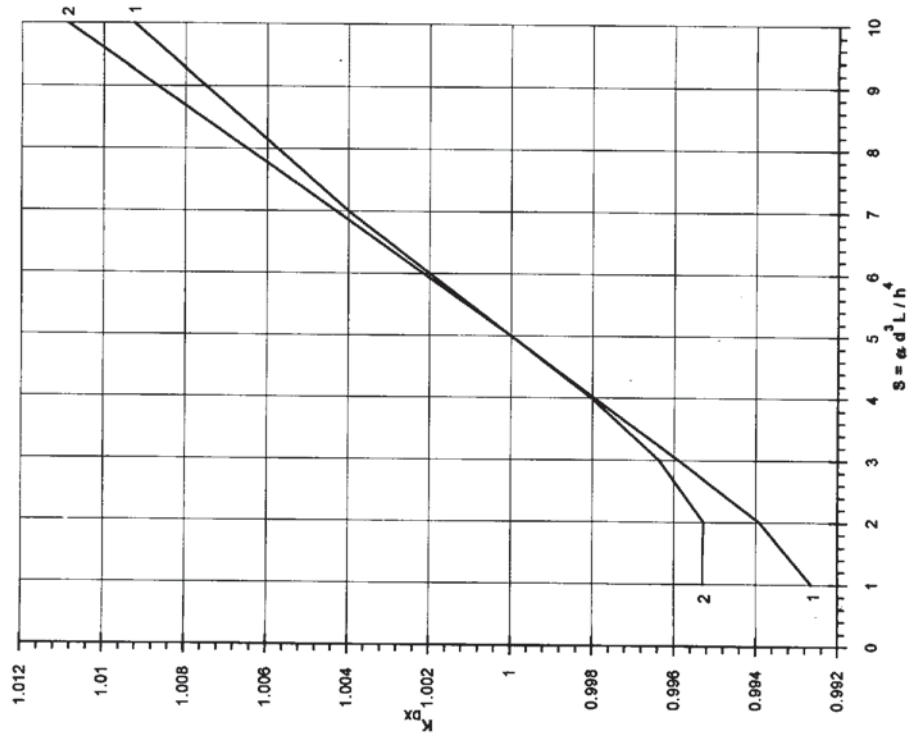
Graph 9.7 K_{Cx} for M_x . Stiffness correction coefficients for Zone C, curves for $I_x/I_y = 1$ and 2.



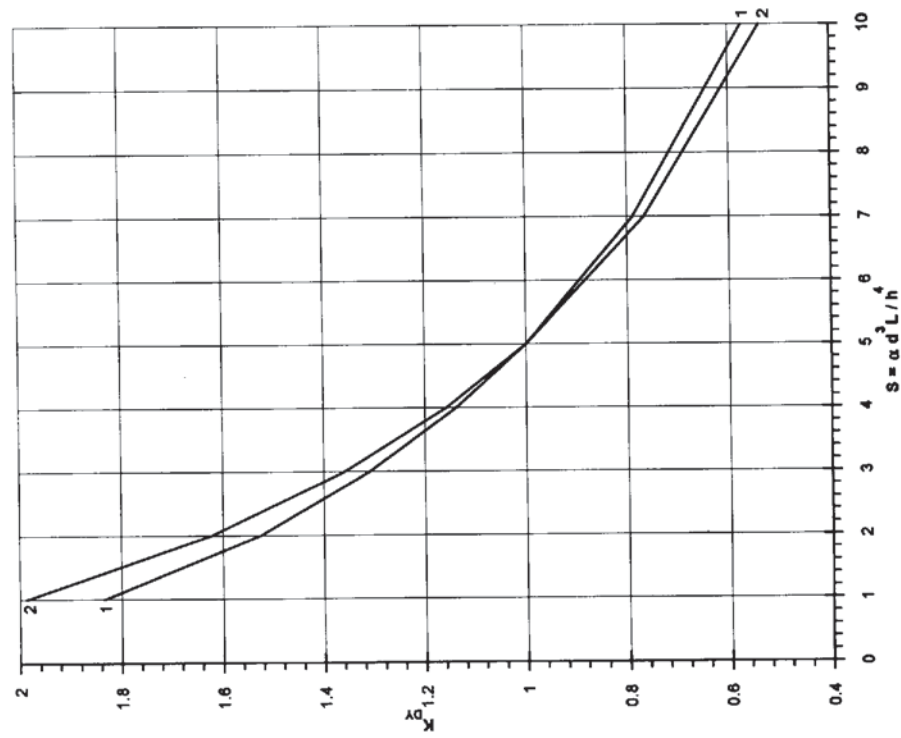
Graph 9.9 K_{Cxy} for M_{xy} . Stiffness correction coefficients for Zone C, curves for $I_x/I_y = 1, 1.4, 1.6$ and 2.



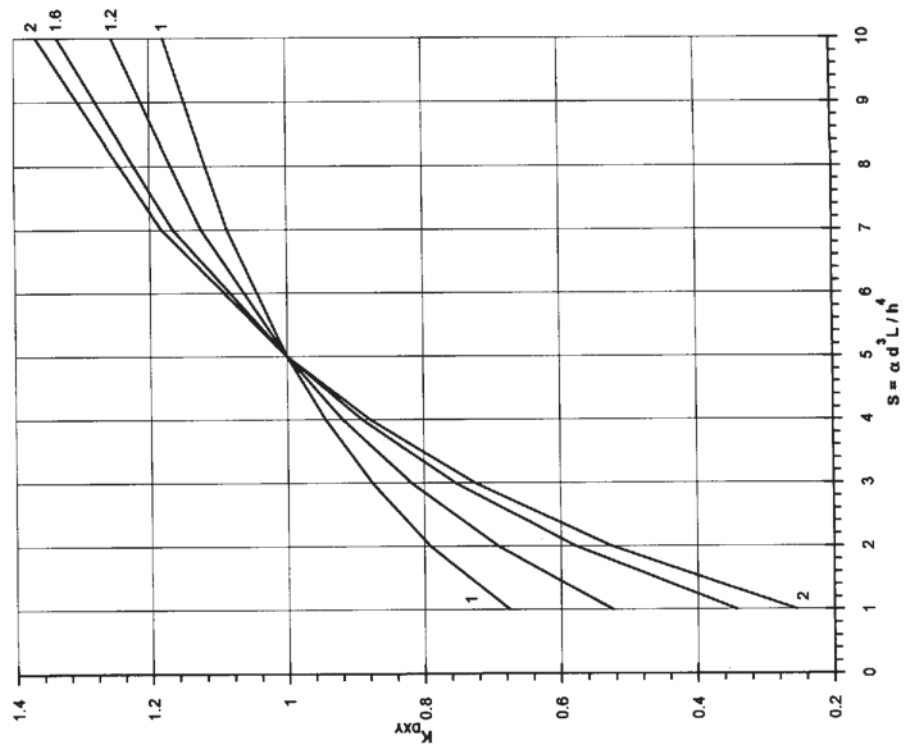
Graph 9.10 K_{Dx} for M_x . Stiffness correction coefficients for Zone D, curves for $I_x/I_y = 1$ and 2.



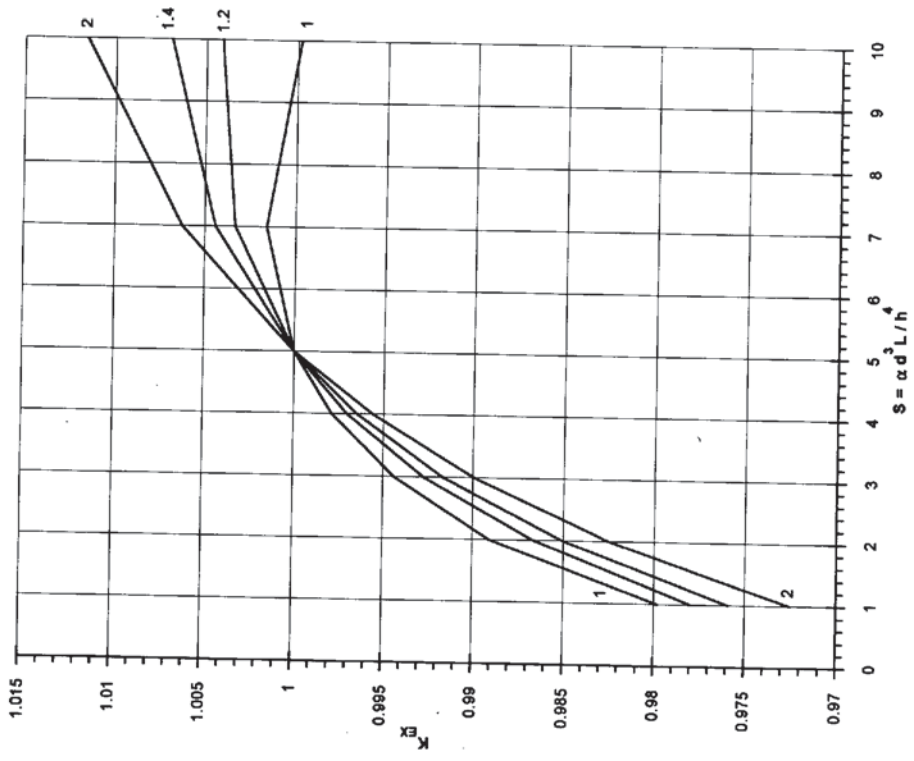
Graph 9.11 K_{Dy} for M_y . Stiffness correction coefficients for Zone D, curves for $I_x/I_y = 1$ and 2.



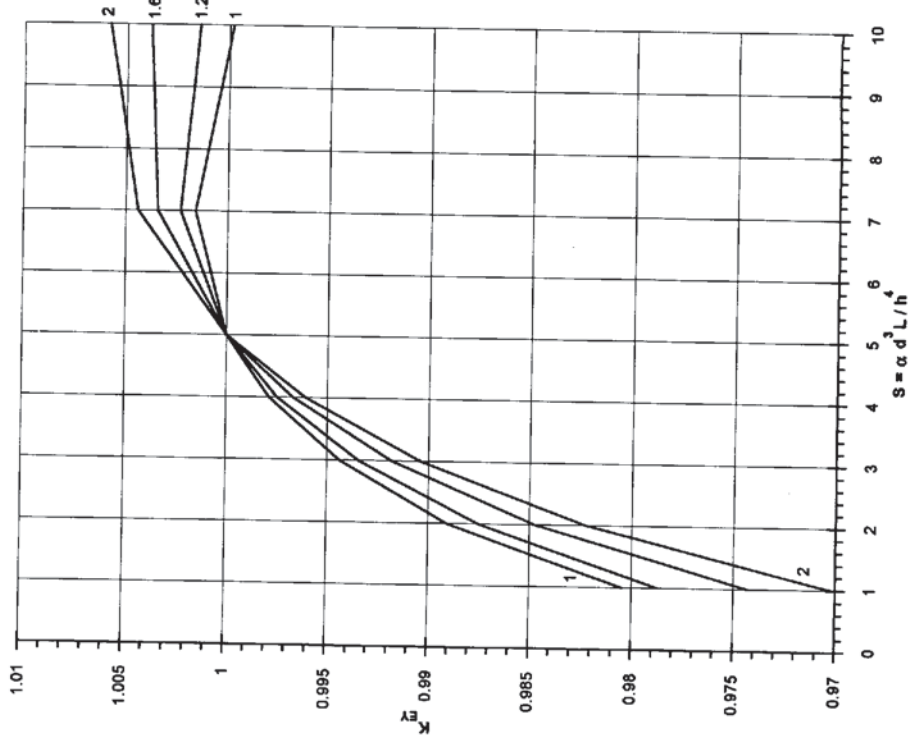
Graph 9.12 K_{Dxy} for M_{xy} . Stiffness correction coefficients for Zone D, curves for $I_x/I_y = 1, 1.2, 1.6$ and 2.



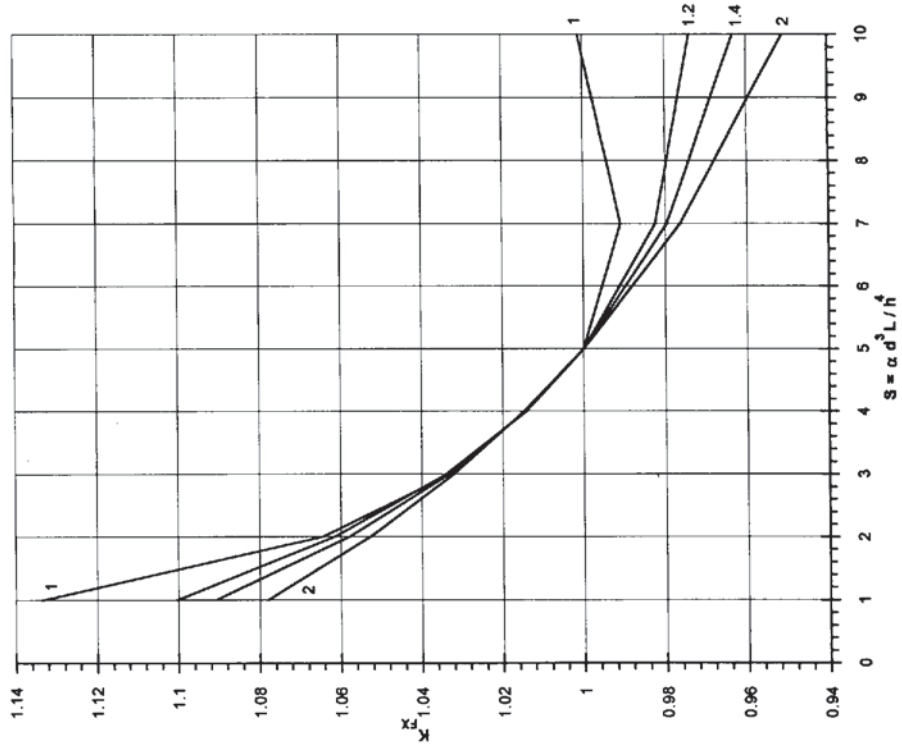
Graph 9.13 K_{Ex} for M_x . Stiffness correction coefficients for Zone E, curves for $l_x/l_y = 1, 1.2, 1.4$ and 2.



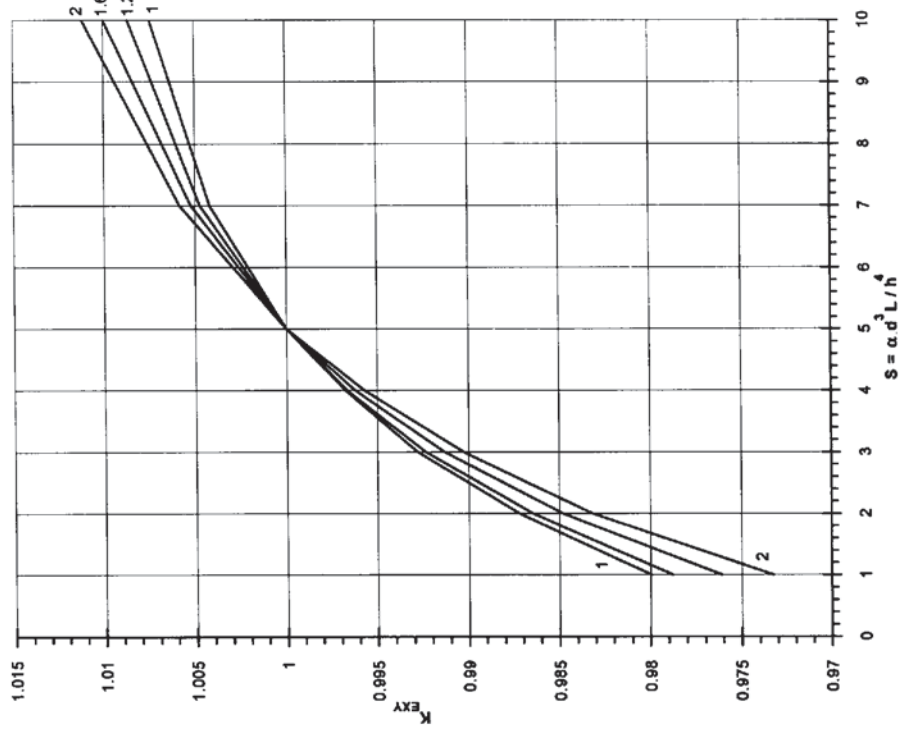
Graph 9.14 K_{Ey} for M_y . Stiffness correction coefficients for Zone E, curves for $l_x/l_y = 1, 1.2, 1.6$ and 2.



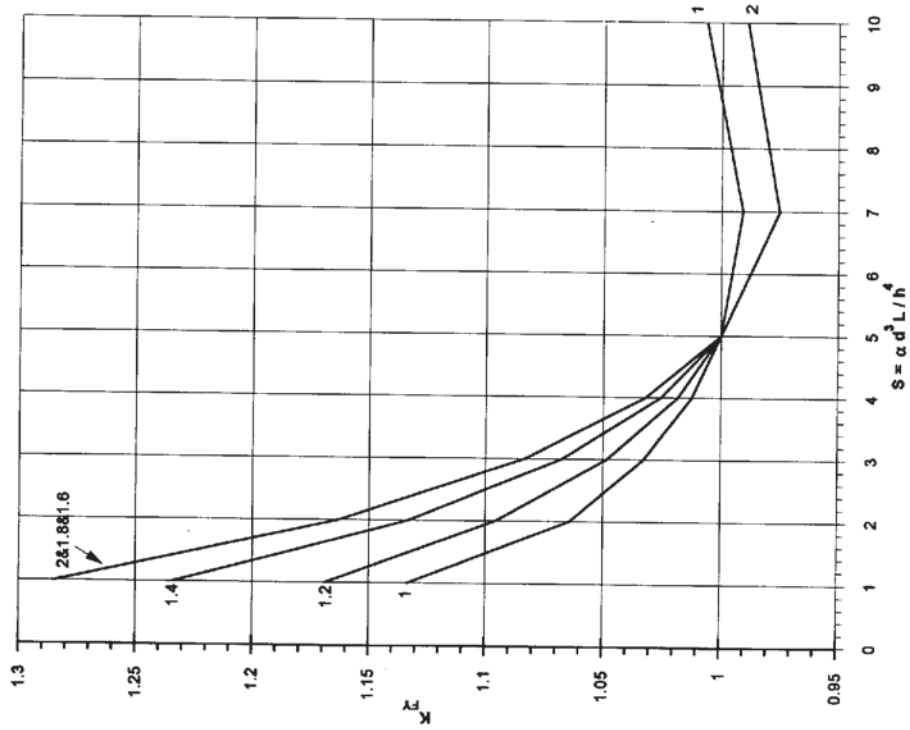
Graph 9.16 K_{Fz} for M_z . Stiffness correction coefficients for Zone F, curves for $I_x/I_y = 1, 1.2, 1.4$ and 2.



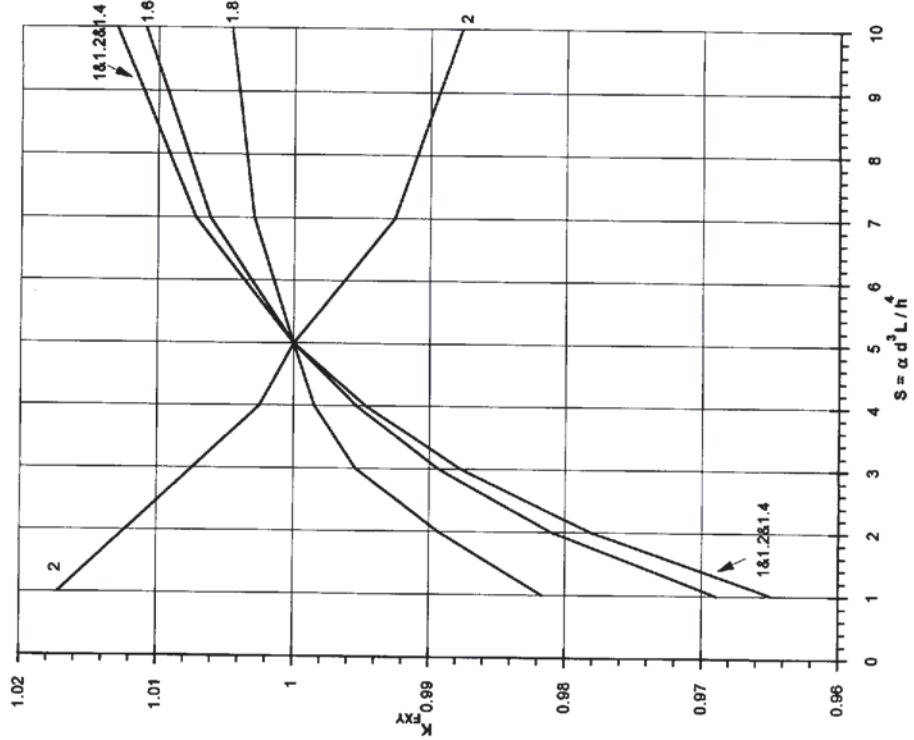
Graph 9.15 K_{Exy} for M_{xy} . Stiffness correction coefficients for Zone E, curves for $I_x/I_y = 1, 1.2, 1.6$ and 2.



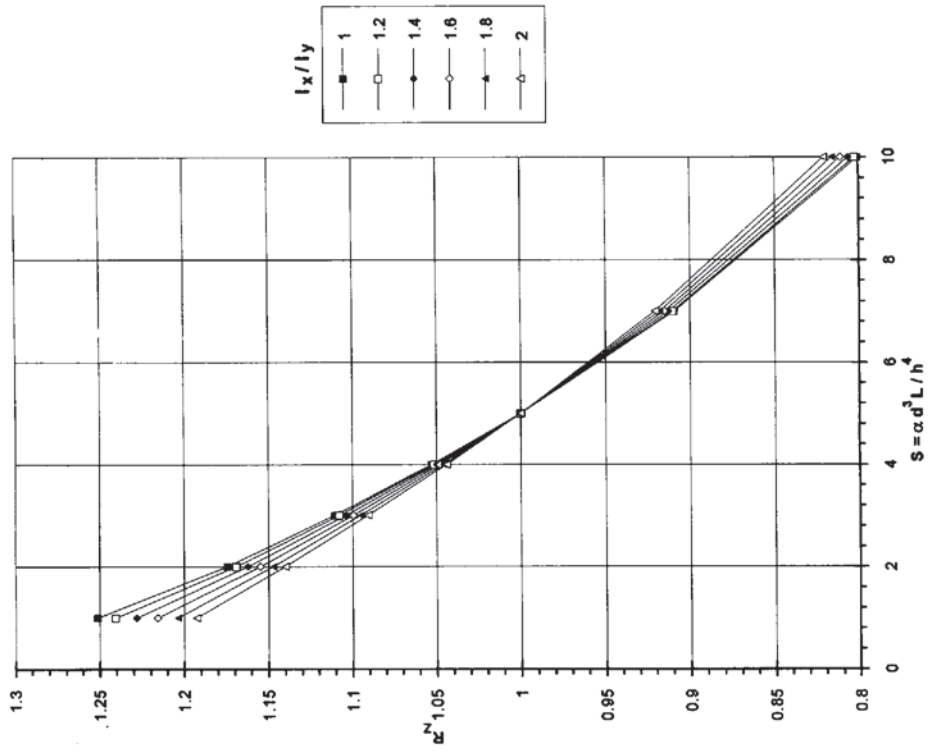
Graph 9.17 K_{Fy} for M_y . Stiffness correction coefficients for Zone F, curves for $l_x/l_y = 1, 1.2, 1.4, 1.6, 1.8$ and 2.



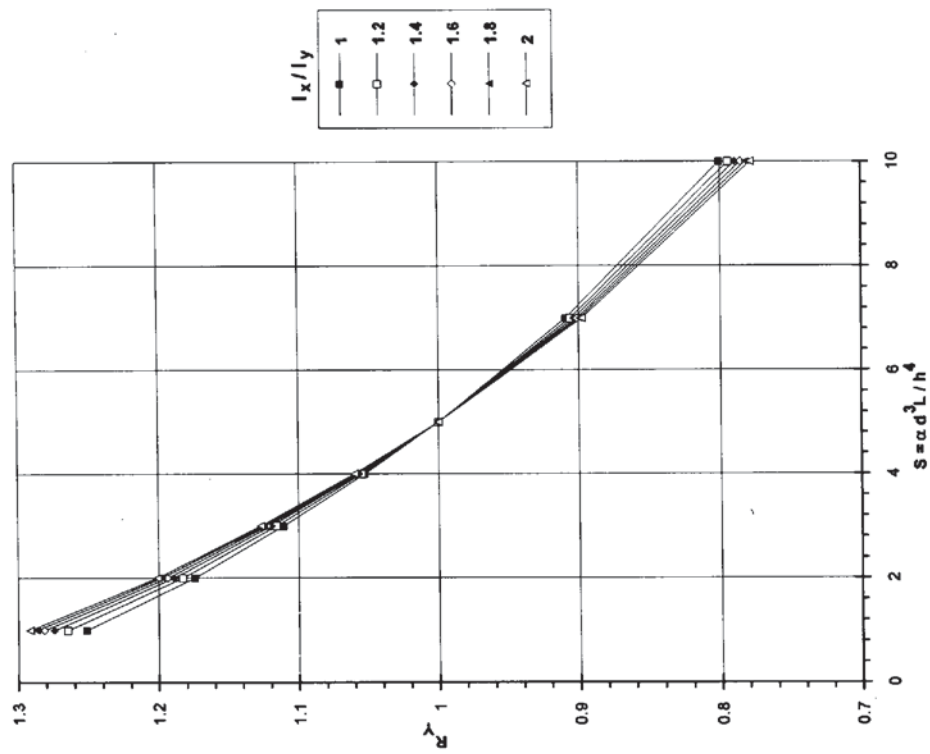
Graph 9.18 K_{Fxy} for M_{xy} . Stiffness correction coefficients for Zone F, curves for $l_x/l_y = 1, 1.2, 1.4, 1.6, 1.8$ and 2.



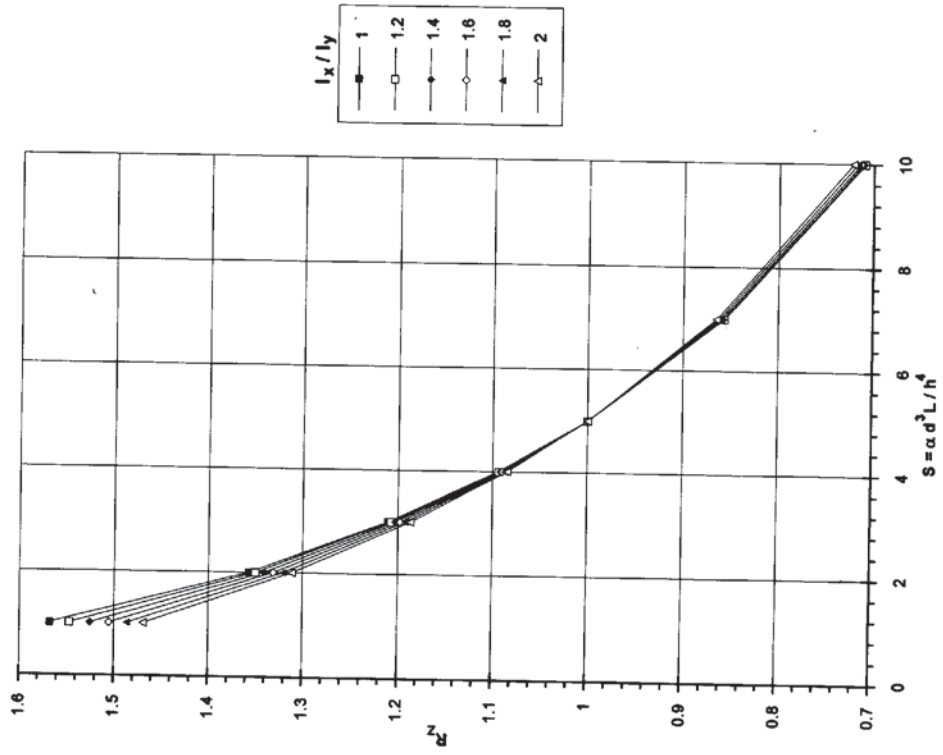
Graph 9.20 R_z for M_z . Stiffness correction coefficients for Column I.



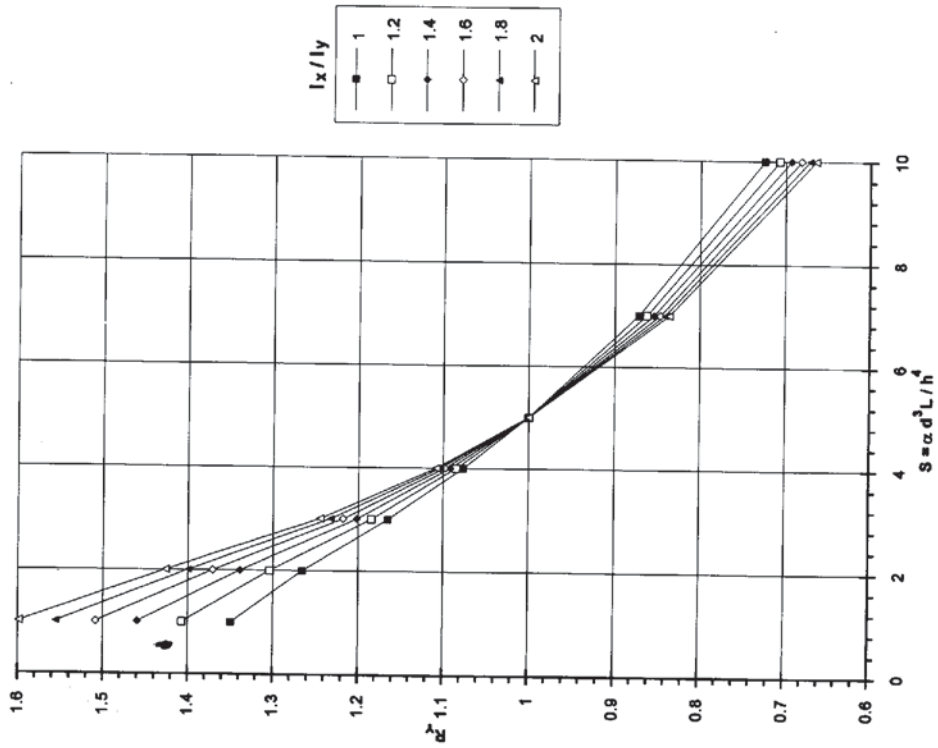
Graph 9.19 R_y for M_y . Stiffness correction coefficients for Column I.



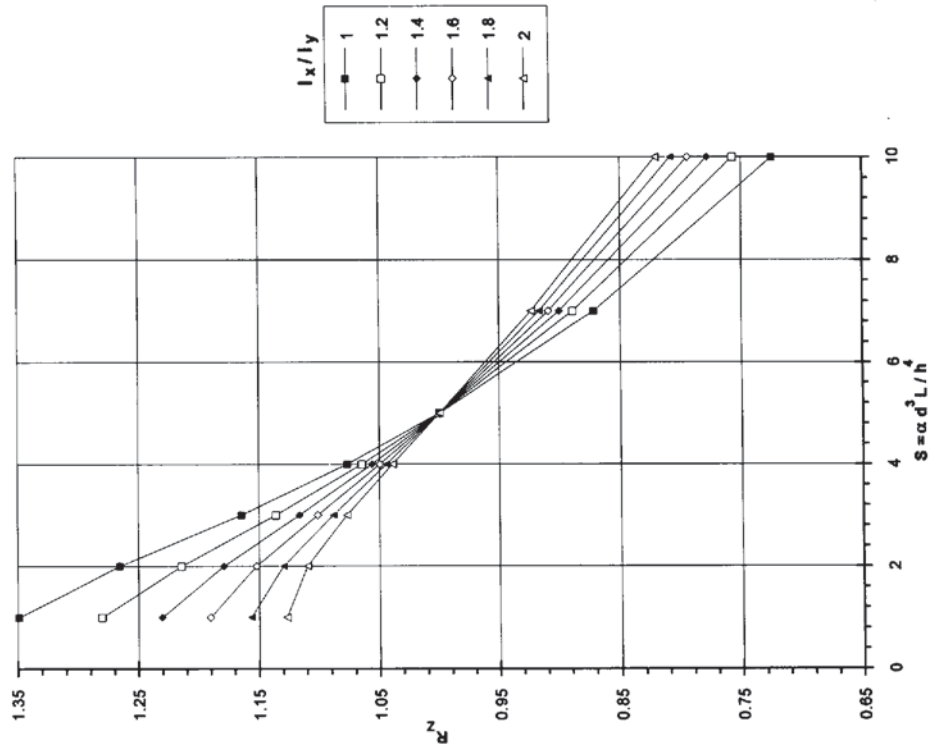
Graph 9.22 R_z for M_z . Stiffness correction coefficients for Column 2.



Graph 9.21 R_y for M_y . Stiffness correction coefficients for Column 2.



Graph 9.24 R_z for M_z . Stiffness correction coefficients for Column 3.



Graph 9.23 R_y for M_y . Stiffness correction coefficients for Column 3.

